پارسا سماعی پسر دهم A

۱۹,۵

۱) ۲ رقم یکان ۲

$(\underbrace{1!+3!}_{1+6=7}+\underbrace{5!+\dots+25!}_{\text{یکان } 0})(\underbrace{2!+4!}_{2+4=6}+\underbrace{6!+\dots+24!}_{\text{یکان } 0}) \Longrightarrow 6\times 7=42$

۲) ۱,۷۵

الف) $\sqrt[4]{(4+\sqrt{5})^{-1}}\sqrt{1+\sqrt{5}} \Longrightarrow \sqrt[4]{\frac{1}{4+\sqrt{5}}}\times\sqrt[4]{8+2\sqrt{5}}=\sqrt[4]{\frac{2(4+\sqrt{5})}{4+\sqrt{5}}}=\sqrt[4]{2}$

ب) $\underbrace{\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2}\right)}_{\frac{1}{\sqrt{2}}}\underbrace{\left(\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}\right)}_{A} \Longrightarrow A^2=3-\cancel{\sqrt{5}}+3+\cancel{\sqrt{5}}-2\left(\frac{\sqrt{9-5}}{2}\right)=2 \Longrightarrow A=\sqrt{2}$

$A<0$ $\qquad \frac{\sqrt{2}}{\sqrt{2}}=-1$

۳) ۱,۷۵

الف) $\frac{\sqrt{8+\sqrt{27}}}{5-\sqrt{6}}-2(\sqrt{6}-1)^{-1}=\frac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}}\times\frac{5+\sqrt{6}}{5+\sqrt{6}}-\frac{2}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\sqrt{2}+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}+1=\sqrt{2}-1$

ب) $\frac{\sqrt{27}-1}{4+\sqrt{3}}+(2-\sqrt{3})^{-1}=\frac{3\sqrt{3}-1}{4+\sqrt{3}}\times\frac{4-\sqrt{3}}{4-\sqrt{3}}+\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\underset{-1}{\sqrt{3}}+2+\sqrt{3}=\cancel{3(1+\sqrt{3})}\ 2\sqrt{3}+1$

۴) ۲

الف) $A\left(\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}\right)-2 \Longrightarrow A^2=\frac{1+\sqrt{3}+\sqrt{3}-1+2\sqrt{2}}{\sqrt{3}-\sqrt{2}}=2(\sqrt{3}+\sqrt{2})^2 \Longrightarrow A=\sqrt{2}(\sqrt{3}+\sqrt{2})=\sqrt{6}+2$ $\qquad \sqrt{6}+2-2=\sqrt{6}$

ب) $\sqrt[4]{\sqrt[3]{2^8}}\times\sqrt[4]{16^2}\times\sqrt{4\sqrt[6]{2}}=\sqrt[12]{2^8}\times\sqrt[4]{2^4}\times\sqrt[12]{2^{13}}=\sqrt[12]{2^{24}}=12$

۵) ۲

$\frac{3^x+3^{x+1}+3^{x+2}+3^{x+3}+3^{x+4}+3^{x+5}}{2^{x-2}+2^{x-1}+2^x+2^{x+1}+2^{x+2}+2^{x+3}}=\frac{3^x\left(\frac{3^6-1}{3-1}\right)}{2^{x-2}\left(\frac{2^6-1}{2-1}\right)}=52 \Longrightarrow \frac{364\times 3^x}{63\times 2^{x-2}}=52=2^2\times 13 \Longrightarrow 364\times 3^x=819\times 2^x$

$\left(\frac{3}{2}\right)^x=\frac{819}{364}=\frac{9}{4} \Longrightarrow x=2$

۶) ۲

$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2=t(2-\sqrt{3})$

$\left\{\begin{array}{l} a=\sqrt[4]{\sqrt{6}-2} \\ b=\sqrt[4]{\sqrt{6}+2}\end{array}\right.$

$((a-b)^2)^2\times((a+b)^2)^2=(a^2-b^2)^4=(\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2})^2=\left[\sqrt{6}-2+\sqrt{6}-2-2\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)}\right]^2$

$[2\sqrt{6}-2\sqrt{2}]^2=4[6+2-2\sqrt{12}]=4[8-4\sqrt{3}]=16[2-\sqrt{3}]=t(2-\sqrt{3}) \Longrightarrow t=16$

۷) ۲

$\left(a+\frac{1}{a}+\sqrt{2}\right)^2\left(a+\frac{1}{a}-\sqrt{2}\right)^2=2^t=\left(a+\frac{1}{a}\right)^2-\left(\frac{\sqrt{2}}{2}\right)^2=a^2+\frac{1}{a^2}+\cancel{2}-\cancel{2}=2-\sqrt{3}+\frac{1}{2-\sqrt{3}}$

$=2-\sqrt{3}+2+\sqrt{3}=4$

$a=\sqrt[4]{7-4\sqrt{3}}=\sqrt{2-\sqrt{3}}$

۸) ۲

$A=\sqrt[5]{4\sqrt[3]{16}\left(\frac{1}{2}\right)^{-\frac{4}{3}}}=\sqrt[15]{2^6\times 2^4}\times\sqrt[3]{2^4}=\sqrt[3]{2^2}\times\sqrt[3]{2^4}=\sqrt[3]{2^6}=4$

$\underbrace{(2A)^{-\frac{1}{3}}}_{8}=\sqrt[3]{\frac{1}{8}}=\frac{1}{2}=0.5$

۹) ۲

$\sqrt[7]{a}=3^3\times a^{\frac{15}{17}} \Longrightarrow a=3^{21}\times a^{15} \Longrightarrow \frac{1}{a^2}=3^3=27 \Longrightarrow \frac{1}{a}=\sqrt{27}$

$\frac{\sqrt{27}-3}{1+\sqrt{3}}=\frac{3\sqrt{3}-3}{1+\sqrt{3}}=\frac{3(\sqrt{3}-1)}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}=\frac{3}{2}(\sqrt{3}-1)^2=9-3\sqrt{3}$

۱۰) ۲

$\sqrt{x+a}-\sqrt{x-4}=2$

$\sqrt{x+a}+\sqrt{x-4}-2=A \qquad (\sqrt{x+a}-\sqrt{x-4})\times(\sqrt{x+a}-\sqrt{x-4})=2A+4$

$\cancel{x}+a-\cancel{x}+\cancel{4}=2A+\cancel{4} \Longrightarrow A=\frac{a}{2}$