

$$(1! + 3! + 5! + \dots + 2001!) (2! + 4! + 6! + \dots + 2002!) = x$$

$$\underbrace{(1 + 3 + 5 + \dots)}_{\text{مجموعه ۱}} \underbrace{(2 + 4 + 6 + \dots)}_{\text{مجموعه ۲}} = x \Rightarrow \boxed{2 = x}$$

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الف) $\sqrt[4]{(4+\sqrt{5})^{-1}} \times \sqrt[4]{1+\sqrt{5}}$
 $\sqrt[4]{\frac{1}{4+\sqrt{5}}} \times \sqrt[4]{1+\sqrt{5}} = \sqrt[4]{\frac{1+\sqrt{5}}{4+\sqrt{5}}} = \sqrt[4]{\frac{1}{2}} = \boxed{\frac{1}{\sqrt{2}}}$

ب) $\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}+2}\right) (\sqrt{3}-\sqrt{5} - \sqrt{2+\sqrt{5}}) = \left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}+2} \times \frac{\sqrt{1}-2}{\sqrt{1}-2}\right) (\sqrt{3}-\sqrt{5} - \sqrt{2+\sqrt{5}}) = \frac{3\sqrt{2}}{4} (\sqrt{3}-\sqrt{5} - \sqrt{2+\sqrt{5}})$
 $= \frac{-2}{4} = \boxed{-1}$

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الف) $\frac{\sqrt{8}+\sqrt{24}}{5-\sqrt{5}} - 2(\sqrt{5}-1)^{-1}$

$$\frac{\sqrt{8}+\sqrt{24}}{5-\sqrt{5}} \times \frac{5+\sqrt{5}}{5+\sqrt{5}} - \frac{2}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{9\sqrt{3}+19\sqrt{5}}{19} - \frac{2+\sqrt{5}}{2}$$

$$\sqrt{3}+\sqrt{2}-1-\sqrt{5} = \boxed{-1+\sqrt{2}}$$

ب) $\frac{\sqrt{24}-1}{4+\sqrt{3}} + (2-\sqrt{3})^{-1}$

$$\frac{\sqrt{24}-1}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} + \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{-12+13\sqrt{3}}{13} + \frac{2+\sqrt{3}}{1}$$

$$\Rightarrow -1+\sqrt{3}+2+\sqrt{3} = \boxed{1+2\sqrt{3}}$$

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الف) $\frac{\sqrt{1+\sqrt{3}} + \sqrt{3-1}}{\sqrt{3}-\sqrt{2}} - 2$

$$\frac{\sqrt{1+\sqrt{3}} + \sqrt{3-1}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} - 2 = \frac{(1+\sqrt{3}) + (3-1)}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} - 2 = \frac{3+\sqrt{6}}{1} - 2 = \boxed{\sqrt{6}+1}$$

ب) $\sqrt[4]{2^8} \times \sqrt[4]{162} \times \sqrt[4]{8\sqrt{2}}$

$$\sqrt[4]{2^8} \times \sqrt[4]{2^5 \times 3^3} \times \sqrt[4]{2^3 \times 2} = 2^2 \times 2^{5/4} \times 3^{3/4} \times 2^{1/2} = 2^2 \times 2^{3/4} \times 3^{3/4} = 2^{11/4} \times 3^{3/4}$$

$$\Rightarrow 2 \times 2^2 \times 3 = \boxed{12}$$

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$$\frac{2^x + 2^{x+1} + 2^{x+2} + 2^{x+3} + 2^{x+4} + 2^{x+5}}{2^{x-2} + 2^{x-1} + 2^x + 2^{x+1} + 2^{x+2} + 2^{x+3}} = \frac{2^x(1+2+2^2+2^3+2^4+2^5)}{2^{x-2}(1+2+2^2+2^3+2^4+2^5)} = \frac{2^x(1+2+4+8+16+32)}{2^{x-2}(1+2+4+8+16+32)} = \frac{2^x(63)}{2^{x-2}(63)} = 2^2 = 4$$

$$2^x(63) = \frac{1}{4} \times 4 \times 2^x(63) \Rightarrow 2^x \times \frac{1}{4} = 2^x \Rightarrow \boxed{x=2}$$

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$$(a^r + b^r - rab)^r (a^r + b^r + rab)^r = (a-b)^r (a+b)^r = (a^r - b^r)^r = (\sqrt{y-r} - \sqrt{y+r})^r = (\sqrt{y-r} + \sqrt{y+r} - 2\sqrt{y})^r$$

$$\Rightarrow (2\sqrt{y} - 2\sqrt{y})^r = 2^r + 1 - 12\sqrt{y} = 12(y - \sqrt{y}) = 12(r - \sqrt{r}) \Rightarrow \boxed{t = 12}$$

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$$a = \sqrt[r]{y - r\sqrt{y}} = \sqrt[r]{\sqrt{r} - \sqrt{r}}$$

$$(a + \frac{1}{a} + \sqrt{r})^r (a + \frac{1}{a} - \sqrt{r})^r = ((a + \frac{1}{a})^r - r)^r = (a^r + \frac{1}{a^r} + r - r)^r$$

$$a^r + \frac{1}{a^r} + r = \sqrt{y - r\sqrt{y}} + \frac{1}{\sqrt{y - r\sqrt{y}}} \times \frac{\sqrt{y + r\sqrt{y}}}{\sqrt{y + r\sqrt{y}}} + r = \sqrt{y - r\sqrt{y}} + \sqrt{y + r\sqrt{y}} + r = 12 = r^t \Rightarrow \boxed{t = 6}$$

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$$A = \sqrt[r]{y\sqrt{12}} \left(\frac{1}{r}\right)^{-\frac{r}{2}} = \sqrt[r]{y^{1/2}} \times \sqrt[r]{y^r} = \sqrt[r]{y^{r/2}} = r$$

$$(rA)^{-\frac{1}{r}} = (r^r)^{-\frac{1}{r}} = \frac{1}{r}$$

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$$\sqrt[r]{a} = r \times \sqrt[r]{a^{1/r}} \Rightarrow \sqrt[r]{r^{r/r} \times a^{1/r}} = \sqrt[r]{r \times a^{1/r}}$$

$$\Rightarrow a = r^{r/r} \times a^{1/r} \Rightarrow a^{1/r} = r^{-r/r} \Rightarrow a = \frac{1}{r^{r/r}}$$

$$\frac{\sqrt{r^r} - r}{1 + \sqrt{r}} \times \frac{1 - \sqrt{r}}{1 - \sqrt{r}} = \frac{-r(r - \sqrt{r})}{-r} = \boxed{r(r - \sqrt{r})}$$

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$$\left. \begin{aligned} \sqrt{x+a} - \sqrt{x-k} &= r \\ \sqrt{x+a} + \sqrt{x-k} &= A \end{aligned} \right\} \begin{aligned} x+a - x+k &= rA = a+k \\ \Rightarrow A &= \frac{a+k}{r} \end{aligned}$$

$$\sqrt{x+a} + \sqrt{x-k} - r = \frac{a+k}{r} - r = \boxed{\frac{a}{r}}$$

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