۲ ۱

ا، ۱ ۵ ، ۲ بعره ۰

$\left.\begin{matrix} 1 \to 1 \\ 3! \to 6 \end{matrix}\right\}(7)_2$

۲ $\left.\begin{matrix} 2! \to 2 \\ 4! \to 24 \end{matrix}\right\}6 \quad (7)(6) = 42$

۳

۴ — الف) $\sqrt[3]{1+\sqrt{7}} = \sqrt[4]{(1+\sqrt{7})^2} \times \sqrt[4]{(4+\sqrt{7})^{-1}}$

$$\sqrt[4]{\frac{\cancel{(1+\sqrt{7})^2}\ 8+2\sqrt{7}}{4+\sqrt{7}}} = \sqrt[4]{\frac{2\cancel{(4+\sqrt{7})}}{\cancel{4+\sqrt{7}}}} = 2^{\frac{1}{4}}$$

ب)

$$\sqrt{3-\sqrt{5}} = \sqrt{3-2\sqrt{\frac{5}{4}}} = \sqrt{\left(\sqrt{\frac{5}{2}}-\sqrt{\frac{1}{2}}\right)^2} =$$

$\frac{5}{2} \times \frac{1}{2} = \frac{5}{4}$

$$\sqrt{\frac{5}{2}} - \sqrt{\frac{1}{2}} \Rightarrow \sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}} =$$

$$\cancel{\sqrt{\frac{5}{2}}} - \sqrt{\frac{1}{2}} - \cancel{\sqrt{\frac{5}{2}}} - \sqrt{\frac{1}{2}} = -2\sqrt{\frac{1}{2}} =$$

$$-\sqrt{2}$$

$$-\sqrt{2}\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2}\right) = \frac{-2-\sqrt{10}}{\sqrt{10}+2} = -1$$

۳)

الف)

$$= \frac{\sqrt[4]{9}+1}{\sqrt[4]{9}+1} \times \frac{-2}{\sqrt[4]{9}-1} = -2(\sqrt[4]{9}-1)^{-1}$$

$$\Rightarrow \sqrt[4]{4}-1 = -\sqrt[4]{9}-1 = \frac{-2(\sqrt[4]{9}+1)}{\cancel{\sqrt[4]{9}-1}}$$

$$= \frac{-5\sqrt{3}-5+\sqrt{18}+\sqrt{6}}{5-\sqrt{6}} = \frac{\sqrt{18}+\sqrt{27}}{5-\sqrt{6}} + \frac{(-\sqrt{3}-1)(5-\sqrt{6})}{5-\sqrt{6}} \Leftarrow$$

$$= \frac{-5(\sqrt{3}+1)+\sqrt{6}(\sqrt{3}+1)}{5-\sqrt{6}}$$

$$\Rightarrow \frac{(\sqrt{3}+1)(-5+\sqrt{6})}{5-\sqrt{6}} = -\sqrt{3}-1$$

ب)

$$(2-\sqrt{3})^{-1} = \frac{1}{2-\sqrt{3}} = 2+\sqrt{3} \Rightarrow$$

$$= \frac{\sqrt{27}-1}{4+\sqrt{3}} + \frac{(2+\sqrt{3})(4+\sqrt{3})}{4+\sqrt{3}}$$

$$= \frac{\sqrt{27}-1+8+4\sqrt{3}+2\sqrt{3}+3}{4+\sqrt{3}} = \frac{10+9\sqrt{3}}{4+\sqrt{3}}$$

(۴)

الف)

$\sqrt{9}$

ب)

$$\sqrt[4]{\sqrt[3]{2^8}} = \sqrt[12]{2^8} = 2^{\frac{8}{12}} = \cancel{2^{\frac{2}{3}}}$$

$$= 2 \times 2^{\frac{1}{4}} \times 4^{\frac{1}{4}} \times 3 \times$$

$$\times 2^{\frac{3}{12}} \times 2^{\frac{8}{12}} \times 3 \times 2^{\frac{12}{12}} \times 2^{\frac{4}{12}} \times 2^{\frac{3}{12}}$$

(۱۴) $= 3 \times 2^{\frac{12}{12}} \times 2^{\frac{12}{12}}$

$$\sqrt{162} = \cancel{\sqrt[4]{81}} \times \sqrt[4]{2}$$

۳

$$= 2\sqrt[12]{2} = 2\sqrt{6\sqrt{2}} = \sqrt[2]{24\sqrt{2}}$$

$2 \times 2^{\frac{1}{2}}$

$3+9+27+81+243$

(۵)

(۲)

364

$$\frac{3^n(\cancel{1+3+\dots+3^5})}{2^n(\cancel{2^{-2}+2^{-1}+1+2+2^2})} = 52$$

$15/75$

$$2^n \times \cancel{15{,}75 \times 52} = 3^n \times 364 \Rightarrow \frac{2^n}{3^n} = \frac{\cancel{364}}{\cancel{819}} = \frac{4}{9}$$

819

$$2^n = 4 \Rightarrow n = 2$$

$$\left(\sqrt{\sqrt{۶}+۲}-\sqrt{\sqrt{۶}-۲}\right)^۲$$

۶=۱۶

(۶)

$$(۲\sqrt{۶}-۲\sqrt{۲})^۲=۲۴+۸-۸\sqrt{۱۲}$$

$$۳۲-۸\sqrt{۱۲}=$$

$$\sqrt{۶}+۲+\sqrt{۶}-۲-۲\sqrt{۲}=$$

$$ab=\sqrt{۲}$$

$$(a^۲+b^۲-۲ab)=(a-b)^۲$$

$$(a^۲+b^۲+۲ab)=(a+b)^۲$$

$$\Rightarrow (a+b)^۲(a-b)^۲=(a^۲-b^۲)^۲$$

$$(a-b)^۲(a+b)^۲=(a^۲-b^۲)$$

$$\left(\sqrt[۲]{\sqrt{۶}+۲}\,\sqrt[۲]{\sqrt{۶}-۲}\right)^۴=$$

$$\left(\sqrt{۶-۴}\right)^۴=\left(۲^{\frac{۱}{۲}}\right)^۴=۴$$

$$\frac{۱-\sqrt{۳}}{۲}\qquad a^۲+\frac{۱}{a^۲}$$

$$\sqrt[۴]{۷-۴\sqrt{۳}}=\sqrt[۴]{۷-۲\sqrt{۱۲}}=\sqrt[۴]{(\sqrt{۳}-\sqrt{۴})^۲}$$

(۷)

$$\Rightarrow \sqrt[۲]{۲-\sqrt{۳}}=\sqrt[۲]{۲-۲\sqrt{\frac{۳}{۴}}}=\sqrt[۲]{\left(\sqrt{\frac{۳}{۴}}-\sqrt{\frac{۱}{۴}}\right)^۲}=$$

$$\sqrt{\frac{۳}{۴}}-\frac{۱}{۲}=A$$

$$\frac{۴-۲\sqrt{۳}}{۴}\qquad \frac{\sqrt{۳}-۱}{۲}+\frac{۲}{\sqrt{۳}+۱}$$

$$\left(\left(a+\frac{۱}{a}\right)^۲-۲\right)^۲\Rightarrow a^۲+\frac{۱}{a^۲}=\left(\frac{۲-\sqrt{۳}}{۲}+\frac{۱}{۲-\sqrt{۳}}\right)^۲$$

$$\left(\frac{\sqrt{۳}-۱}{۲}+\frac{۲}{\sqrt{۳}-۱}\right)$$

$$\frac{\sqrt{۳}-۱}{۲}+\frac{۲}{\sqrt{۳}-۱}=\frac{\sqrt{۳}-۱}{۲}+\frac{۲(\sqrt{۳}+۱)}{۲}=\frac{۳\sqrt{۳}+۱}{۲}\Rightarrow\left(\frac{۳\sqrt{۳}+۱}{۲}\right)^۲-۲=$$

$$\left(\frac{۸\sqrt{۳}+۸+\sqrt{۳}-۱}{۴}\right)^۲=\left(\frac{۹\sqrt{۳}+۱۶}{۴}\right)$$

$$\frac{۲۷+۱+۹\sqrt{۳}}{۴}-۲=$$

$$a^۴+\frac{۱}{a^۴}$$

(8)

$$A = 2^{3} = 2^{\frac{9}{3}} = \cdots = 2^{\frac{1}{3 \times 5}} \times 2^{\frac{2}{3}} = \sqrt[5]{2^{3} \times 2^{\frac{4}{3}}} \times 2^{\frac{2}{3}}$$

$$\left(2 \times 2\right)^{\frac{1}{3}} \quad \frac{1}{2}$$

(9)

$$\sqrt[7]{a} = 27 \times a^{\frac{15}{7}}$$

$$a^{\frac{1}{7}} = 27 \times a^{\frac{15}{7}} \Rightarrow a^{\frac{1}{7}} \div a^{\frac{15}{7}} = 27$$

$$a^{-\frac{14}{7}} = 27$$

تعریف نشده

$$a^{-2} = 27$$

$$\frac{1}{a^{2}} \Rightarrow 27$$

$$27a^{2} = 1$$

$$a^{2} = \frac{1}{27} \Rightarrow a = \frac{1}{\sqrt{27}} = \left(3\sqrt{3}\right)^{-1}$$

$3\sqrt{3}$

$$\frac{1}{a} - 3 = \sqrt{27} - 3$$

$$\frac{\sqrt{27} - 3}{1 + \sqrt{3}} \Rightarrow \frac{3(\sqrt{3} - 1)}{1 + \sqrt{3}} = \frac{3(\sqrt{3}-1)^{2}}{-2}$$

$$\left(\sqrt{27} - 3\right)\left(1 - \sqrt{3}\right)$$

$$3\sqrt{3} - 3 - 3\sqrt{3} + 3\sqrt{3}$$

$$-2$$

$$a \quad \frac{3(3 + 1 - 2\sqrt{3})}{-2} = \frac{12 - 6\sqrt{3}}{-2}$$

(10)

$$\sqrt{x + A} - \sqrt{x - 4} = 2$$

$$\sqrt{x + A} + \sqrt{x - 4} - 2 = ?$$

b

$$2b = x + A - x + 4$$

$$2b = A + 4 \Rightarrow b = \frac{A + 4}{2}$$

PB