برگ امتحانی

ابرسین ماروچیان — مقطع: دوم پسر A — شماره کلاس: ۲۱

۲۰ عالی نوشتی!

سوال ۱ (۲)

$$\frac{(1!+3!+5!+\cdots+25!)}{(1+5+\cdots+45)} \Big/ \frac{(1!+3!+5!+\cdots+25!)}{(2+4+\cdots+46)} = 23$$

رقم یکان = ۳ ✓

سوال ۲ (۲)

الف) $\left(\frac{1}{4+\sqrt{7}}\right)^{\frac{1}{4}} \times (1+\sqrt{7})^{\frac{1}{4}} \times (1+\sqrt{7})^{\frac{1}{4}} = \left(\frac{1}{4+\sqrt{7}} \times 2(4+\sqrt{7})\right)^{\frac{1}{4}} = 2^{\frac{1}{4}} = \sqrt[4]{2}$ ✓

ب) $\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}\right)$ — a عددی است

$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}(\sqrt{5}+\sqrt{2})} = \frac{1}{\sqrt{2}} \times a$

$a^2 = 3-\sqrt{5}+3+\sqrt{5}-2\sqrt{4} = 6-4 = 2 \Rightarrow a = \pm\sqrt{2}$ ✓ (a منفی است)

$\Rightarrow \frac{1}{\sqrt{2}} \times -\sqrt{2} = -1$ ✓

سوال ۳ (۲)

الف) $\frac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}} - \frac{2}{\sqrt{3}-1} \rightarrow \frac{(2\sqrt{2}+3\sqrt{3})(5+\sqrt{6})}{25-6} - \frac{2(\sqrt{3}+1)}{3-1}$

$\rightarrow \frac{10\sqrt{2}+15\sqrt{3}+4\sqrt{3}+9\sqrt{2}}{19} - \sqrt{3}+1 \rightarrow \frac{19\sqrt{2}+19\sqrt{3}-19\sqrt{3}-19}{19} = \frac{19\sqrt{2}-19}{19} = \sqrt{2}-1$ ✓

ب) $\frac{\sqrt{2}-1}{4+\sqrt{3}} + (2-\sqrt{3})^{-1} = \frac{\sqrt{3}-1}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} + \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{13\sqrt{3}-13}{13} + 2+\sqrt{3}$

$\rightarrow \sqrt{3}-1+2+\sqrt{3} = 1+2\sqrt{3}$ ✓

سوال ۴ (۲)

الف) $\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} = A \Rightarrow A^2 = \frac{\sqrt{3}+1+\sqrt{3}-1+2\sqrt{3-1}}{\sqrt{3}-\sqrt{2}} = \frac{2\sqrt{3}+2\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{2(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = 2(\sqrt{3}+\sqrt{2})^2 = A^2 = \sqrt{2}(\sqrt{3}+\sqrt{2}) = \sqrt{6}+2$

$A-2 = \sqrt{6}+2-2 = \sqrt{6}$ ✓

ب) $2^{\frac{8}{12}} \times 3 \times 2^{\frac{3}{12}} \times 2^{\frac{13}{12}} = 3 \times 2^{\frac{24}{12}} = 3 \times 2^2 = 3 \times 4 = 12$ ✓

برگه امتحانی

تاریخ امتحان:
مدرسه / آموزشگاه:
نام و نام خانوادگی:

سوال ۵ (۲)

$$\frac{a(q^n-1)}{q-1} \Rightarrow \frac{3^{2x}(1+3+3^2+3^3+3^4+3^5)}{3^{2x-2}(1+3+3^2+3^3+3^4+3^5)} = ?$$

$$\frac{3^{2x}\times\frac{(3^6-1)}{3-1}}{3^{2x-2}\times\frac{(3^6-1)}{3-1}} = \frac{3^{2x}\times 3^6}{3^{2x-2}\times 3^4} = \frac{3^{2x}\times 3^2}{3^{2x-2}\times 3^2} \Rightarrow \frac{3^{2x-2}}{3^{2x-2}}\times 3^2 = 3^2 = \left(\frac{3}{2}\right)^{2x-2} = 1$$

$$\Rightarrow 2x-2=0 \Rightarrow \boxed{2x=2} \checkmark$$

سوال ۶ (۲)

$$(a-b)^4\times(a+b)^4 = (a^2-b^2)^4 = t(2-\sqrt{3})$$

$$\Rightarrow \left(\left(\sqrt{6-2\sqrt{6-4}}\right)^2\right)^2 = (2\sqrt{6}-2\sqrt{6-4})^2 =$$

$$\rightarrow (4\sqrt{6}-2\sqrt{2})^2 = 4(\sqrt{3}-\sqrt{2})^2 = 4(6-2\sqrt{12}) = 4(8-4\sqrt{3}) =$$

$$16(2-\sqrt{3}) \Rightarrow \boxed{t=16} \checkmark$$

سوال ۷

$$\left(\left(a+\frac{1}{a}\right)^2-2\right)^2 = \left(a^2+\frac{1}{a^2}+2-2\right)^2 \rightarrow a^4+\frac{1}{a^4}+2 = 7-4\sqrt{3}+\frac{1}{7-4\sqrt{3}}+2$$

$$9-6\sqrt{3}+\frac{7+4\sqrt{3}}{49-48} = 9-6\sqrt{3}+7+4\sqrt{3} = 16 = 2^t \quad (2)$$

$$16 = 2^t \Rightarrow \boxed{t=4} \checkmark$$

سوال ۸ (۲)

$$A = \sqrt[3]{4\sqrt{16}}\left(\frac{1}{2}\right)^{-\frac{2}{3}} \rightarrow \sqrt[3]{2^{10}}\,(2)^{\frac{2}{3}} \rightarrow 2^{\frac{2}{3}}\times 2^{\frac{2}{3}} = 2^{\frac{4}{3}} = 4$$

$$(2A)^{-\frac{1}{3}} \rightarrow (8)^{-\frac{1}{3}} \rightarrow (2^3)^{-\frac{1}{3}} \rightarrow 2^{-1} \rightarrow \boxed{\frac{1}{2}} \checkmark$$

سوال ۹ (۲)

$$\sqrt[7]{a} = 27\sqrt[7]{a^{15}} \rightarrow a^{\frac{1}{7}} = 27\times a^{\frac{15}{7}} \rightarrow \frac{a^{\frac{1}{7}}}{a^{\frac{15}{7}}} = 27$$

$$a^{-\frac{14}{7}} = 27 \rightarrow a^{-2} = 27 \rightarrow \left(\frac{1}{a}\right)^2 = 27 \rightarrow \frac{1}{a} = \sqrt{27} \rightarrow a = \sqrt{\frac{1}{27}}$$

$$\left(\frac{1}{a}-3\right) \rightarrow \sqrt{27}-3 \rightarrow 3\sqrt{3}-3 \rightarrow 3(\sqrt{3}-1) \quad \left|\ \frac{3(\sqrt{3}-1)}{\sqrt{3}+1}\times\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{3(3+1-2\sqrt{3})}{3-1}\right.$$

$$\frac{3(4-2\sqrt{3})}{2} = \frac{6(2-\sqrt{3})}{2} = 3(2-\sqrt{3}) = \boxed{6-3\sqrt{3}} \checkmark$$

سوال ۱۰ (۲)

$$\left(\sqrt{x+9}-\sqrt{x-4}\right)\left(\sqrt{x+9}+\sqrt{x-4}\right) = 2\left(\sqrt{x+9}+\sqrt{x-4}\right)$$

$$x+9-x+4 = 2\sqrt{x+9}+2\sqrt{x-4} \Rightarrow 9 = 2\sqrt{x+9}+2\sqrt{x-4}-4$$

$$x = 2\left(\sqrt{x+9}+\sqrt{x-4}-2\right) \Rightarrow \sqrt{x+9}+\sqrt{x-4}-2 = \boxed{\frac{9}{2}} \checkmark$$