

سوال ۱

$$(1! + 3! + 5! + \dots + 45!) = 2$$

$$(1 + 3 + 5 + \dots + 45) = 7 \times 9 = 63$$

رقم یکان = ۳

سوال ۲

$$\left(\frac{1}{4+\sqrt{5}}\right)^{\frac{1}{4}} \times (1+\sqrt{5})^{\frac{1}{4}} \times (1+\sqrt{5})^{\frac{1}{4}} = 2$$

$$\left(\frac{1}{4+\sqrt{5}} \times 2(4+\sqrt{5})\right)^{\frac{1}{4}} = 2^{\frac{1}{4}} = \sqrt[4]{2}$$

$$\Rightarrow \left(\frac{\sqrt{2} + \sqrt{5}}{\sqrt{10} + 2}\right) (\sqrt{3} - \sqrt{5} - \sqrt{3+5}) = \frac{\sqrt{2} + \sqrt{5}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 9$$

$$= 3 - \sqrt{5} + 3 + \sqrt{5} - 2\sqrt{4} = 6 - 4 = 2 \Rightarrow \frac{1}{\sqrt{2}} \times \sqrt{2} = 1$$

سوال ۳

$$\frac{2\sqrt{2} + 3\sqrt{3}}{2 - \sqrt{5}} - \frac{2}{\sqrt{3} - 1} \Rightarrow \frac{(2\sqrt{2} + 3\sqrt{3})(2 + \sqrt{5})}{2 \cdot 5 - 5} - \frac{2(\sqrt{3} + 1)}{3 - 1}$$

$$\Rightarrow \frac{10\sqrt{2} + 15\sqrt{3} + 4\sqrt{6} + 9\sqrt{5}}{10} - \sqrt{3} + 1 \Rightarrow \frac{19\sqrt{2} + 19\sqrt{3} - 19\sqrt{5} - 19}{19} = \frac{19\sqrt{2} - 19}{19} = \sqrt{2} - 1$$

$$\frac{2\sqrt{2} - 1}{4 + \sqrt{5}} + (2 - \sqrt{3})^{-1} = \frac{2\sqrt{2} - 1}{4 + \sqrt{5}} \times \frac{4 - \sqrt{5}}{4 - \sqrt{5}} + \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{8\sqrt{2} - 4 - \sqrt{10} + \sqrt{5}}{15} + \frac{2 + \sqrt{3}}{13}$$

$$\Rightarrow \sqrt{3} - 1 + 2 + \sqrt{3} = 1 + 2\sqrt{3}$$

سوال ۴

$$\frac{\sqrt{1+\sqrt{5}} + \sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{2}-5}} = A \Rightarrow A^4 = \frac{\sqrt{5}+1 + \sqrt{5}-1 + 2\sqrt{5}}{\sqrt{5}-5} = \frac{2\sqrt{5}+2\sqrt{5}}{\sqrt{5}-5}$$

$$= \frac{2(\sqrt{5}+\sqrt{5})}{(\sqrt{5}-5)} \times \frac{(\sqrt{5}+\sqrt{5})}{(\sqrt{5}+\sqrt{5})} = \frac{2(\sqrt{5}+\sqrt{5})^2}{(\sqrt{5}-5)} = A = \sqrt{2}(\sqrt{5}+\sqrt{5}) = \sqrt{5} + \sqrt{5}$$

$$A - 2 = \sqrt{5} + \sqrt{5} - 2 = 2\sqrt{5}$$

$$\Rightarrow 2^{\frac{1}{12}} \times 3^{\frac{1}{12}} \times 2^{\frac{1}{12}} \times 2^{\frac{1}{12}} = 3 \times 2^{\frac{1}{12}} = 3 \times 2^{\frac{1}{12}} = 3 \times 2 = 12$$

سوال ۵

$$9 \left(\frac{9^x - 1}{9 - 1} \right) \Rightarrow 3^{2x} (1 + 3 + 3^2 + 3^3 + 3^4 + 3^5)$$

$$3^{2x} \times \frac{(3^6 - 1)}{3 - 1} = \frac{3^{2x-2} (1 + 3 + 3^2 + 3^3 + 3^4 + 3^5)}{2} = 2$$

$$\frac{3^{2x-2} (3^6 - 1)}{2 \times (3 - 1)} = \frac{3^{2x-2} \times 3^5 \times 2}{2 \times 2} = \frac{3^{2x-2} \times 3^5}{2} \Rightarrow \frac{3^{2x-2}}{2} \times 3^5 = 2 \Rightarrow \left(\frac{3}{2} \right)^{2x-2} = 1$$

$$\Rightarrow 2x - 2 = 0 \Rightarrow \boxed{2x = 2}$$

سوال ۶

$$(a-b)^4 \times (a+b)^4 = (a^2 - b^2)^4 = t (2 - \sqrt{3})$$

$$\Rightarrow ((\sqrt{6} - 2 - \sqrt{6} - 2)^2)^2 = (2\sqrt{6} - 2\sqrt{6} - 4)^2$$

$$\Rightarrow (4\sqrt{6} - 2\sqrt{2})^2 = 4(\sqrt{3} - \sqrt{2})^2 = 4(6 - 2 - 2\sqrt{6}) = 4(4 - 2\sqrt{6}) = 2$$

$$16(2 - \sqrt{3}) \Rightarrow \boxed{t = 16}$$

سوال ۷

$$\left(\left(a + \frac{1}{a} \right)^2 - 2 \right)^2 = \left(a^2 + \frac{1}{a^2} + 2 - 2 \right)^2 \Rightarrow 9 + \frac{1}{9} + 2 = 7 - \frac{1}{9} \Rightarrow \frac{1}{9} = \frac{1}{9}$$

$$9 - \frac{1}{9} + \frac{7 + \frac{1}{9}}{9 - \frac{1}{9}} = 9 - \frac{1}{9} + 7 + \frac{1}{9} = 16 = 2^4$$

$$16 = 2^4 \Rightarrow \boxed{t = 4}$$

سوال ۸

$$A = \sqrt[5]{4\sqrt{19}} \left(\frac{1}{2} \right)^{-\frac{5}{2}} \rightarrow \sqrt[5]{2^{10}} (2)^{\frac{5}{2}} \rightarrow 2^{\frac{10}{5}} \times 2^{\frac{5}{2}} = 2^{\frac{25}{2}} = 2^{\frac{25}{2}}$$

$$(2A)^{-\frac{1}{2}} \rightarrow (1)^{-\frac{1}{2}} \rightarrow (2^{\frac{25}{2}})^{-\frac{1}{2}} \rightarrow 2^{-\frac{25}{4}} \rightarrow \boxed{\frac{1}{2}}$$

سوال ۹

$$\sqrt{a} = 2\sqrt{a} \Rightarrow \sqrt{a} = 2\sqrt{a} \Rightarrow a^{\frac{1}{2}} = 2\sqrt{a} \Rightarrow \frac{a^{\frac{1}{2}}}{a^{\frac{1}{2}}} = 2\sqrt{a} \Rightarrow 1 = 2\sqrt{a} \Rightarrow a = \frac{1}{4}$$

$$\left(\frac{1}{a} - 3 \right) \rightarrow \sqrt{2} - 3 \rightarrow 3\sqrt{2} - 3 \rightarrow 3(\sqrt{2} - 1) \left| \frac{2(\sqrt{2}-1)}{\sqrt{2}+1} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \frac{2(3+1-2\sqrt{2})}{2-1} \right|$$

$$\frac{3(4-2\sqrt{2})}{2} = \frac{6(2-\sqrt{2})}{2} = 3(2-\sqrt{2}) = \boxed{6 - 3\sqrt{2}}$$

سوال ۱۰

$$(\sqrt{x+a} - \sqrt{x-a})(\sqrt{x+a} + \sqrt{x-a}) = 2(\sqrt{x+a} + \sqrt{x-a})$$

$$x+a - x+a = 2\sqrt{x+a} + 2\sqrt{x-a} \Rightarrow a = 2\sqrt{x+a} + 2\sqrt{x-a} - 4$$

$$a = 2(\sqrt{x+a} + \sqrt{x-a} - 2) \Rightarrow \sqrt{x+a} + \sqrt{x-a} - 2 = \boxed{\frac{a}{2}}$$