

از اینجا که توان اول به بعد یکان صفر است

$$(1! + 3! + 5! + \dots + 2001!) \quad 1 + 4 = 5$$

$$(2! + 4! + 6! + \dots + 2002!) \quad 2 + 24 = 26$$

$$4 \times 5 = 20 \rightarrow 2$$

الف) $(k + \sqrt{u})^{-\frac{1}{r}} \times (1 + \sqrt{u})^{\frac{1}{r}} \Rightarrow (1 + \sqrt{u})^{\frac{1}{r}} = 1 + \sqrt{u} + u = 1 + \sqrt{u} = 2(k + \sqrt{u})$

$$k + \sqrt{u} = \frac{(1 + \sqrt{u})^r}{2} \rightarrow (k + \sqrt{u})^{-\frac{1}{r}} \Rightarrow \left(\frac{(1 + \sqrt{u})^r}{2} \right)^{-\frac{1}{r}} \Rightarrow 2^{\frac{1}{r}} \times (1 + \sqrt{u})^{-\frac{1}{r}}$$

$$\Rightarrow (1 + \sqrt{u})^{-\frac{1}{r}} \times (1 + \sqrt{u})^{\frac{1}{r}} \times 2^{\frac{1}{r}} = \boxed{\frac{2}{\sqrt{r}}}$$

ب) $(\sqrt{k - \sqrt{5}} - \sqrt{k + \sqrt{5}})^2 = (k - \sqrt{5}) + (k + \sqrt{5}) - 2\sqrt{(k - \sqrt{5})(k + \sqrt{5})} = 4 - 2\sqrt{k} = 4 - k = 2$

$$\Rightarrow \sqrt{k - \sqrt{5}} - \sqrt{k + \sqrt{5}} = -\sqrt{2} \rightarrow (-\sqrt{2}) \left(\frac{\sqrt{5} + \sqrt{2}}{2 + \sqrt{5}} \right) = -\frac{2 + \sqrt{10}}{\sqrt{5} + 2} = \boxed{-1}$$

الف) $\frac{\sqrt{2u}}{\sqrt{u}} = \frac{3\sqrt{u}}{\sqrt{u}} \rightarrow \frac{2\sqrt{2} + 3\sqrt{u}}{\sqrt{2} - 5}$

$$\sqrt{4} = \sqrt{u^2} = \sqrt{u} \rightarrow (\sqrt{u} - 1)^{-1} = \frac{u}{\sqrt{u} - 1}$$

$$\Rightarrow \frac{2}{\sqrt{u} - 1} \times \frac{\sqrt{u} + 1}{\sqrt{u} + 1} = 1 + \sqrt{u}$$

$$\frac{2\sqrt{2} + 3\sqrt{u}}{5 - \sqrt{2}} \times \frac{5 + \sqrt{2}}{5 + \sqrt{2}} = \frac{19\sqrt{2} + 19\sqrt{u}}{19} = \sqrt{u} + \sqrt{u}$$

ب) $(\sqrt{2} + \sqrt{u}) - (\sqrt{u} + 1) = \boxed{\sqrt{2} - 1}$

$$\frac{1 - 3\sqrt{u}}{4 + \sqrt{u}} \times \frac{4 - \sqrt{u}}{4 - \sqrt{u}} = \frac{14\sqrt{u} - 13}{14} = \sqrt{u} - 1$$

الف) $\sqrt{u} - \sqrt{2} = \sqrt{2} - 1$

$$\frac{\sqrt{1 + \sqrt{u}} + \sqrt{\sqrt{u} - 1}}{\sqrt{u} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = \frac{\sqrt{2}(\sqrt{u} + 1)}{(\sqrt{u} - 1)(\sqrt{2} + 1)} = \sqrt{2}(\sqrt{u} + 1)$$

$$\sqrt{2}(\sqrt{u} + 1) - 2 = \boxed{2\sqrt{u} + \sqrt{2} - 2}$$

ب) $\sqrt[12]{2^8} \times \sqrt[12]{2^4} \times \sqrt[12]{2^4} = \sqrt[12]{2^{16}} = \sqrt[12]{2^4} = \sqrt[3]{2} = \sqrt[3]{2^1} = \sqrt[3]{2}$

$$u \times k = \boxed{12}$$

عبارت های فاکتور لای شده مانند، لای نشده مانند

$$\frac{u^x(1 + u + u^2 + u^3 + u^4 + u^5)}{u^{x-2}(1 + u + u^2 + u^3 + u^4 + u^5)} = \frac{u^x \times \frac{u^6 - 1}{u - 1}}{u^{x-2} \times \frac{u^6 - 1}{u - 1}} = \frac{u^x}{u^{x-2}} = u^2$$

$$\frac{u^4 k}{4u} = \frac{52}{4} \times \frac{u^2}{u^{x-2}} = \frac{52}{4} \times k \times \left(\frac{u}{u} \right)^x = \frac{13}{1} \left(\frac{u}{u} \right)^x = 52$$

$$\Rightarrow \left(\frac{u}{u} \right)^x = 52 \times \frac{4}{13 \times 1} = \frac{4}{1} \Rightarrow \left(\frac{u}{u} \right)^x = \frac{4}{1} \Rightarrow \boxed{x = 2}$$

$$\sqrt[r]{r} - r = a \quad b = \sqrt[r]{r+r} \quad (\text{cancel out } r)$$

$$(a-b)^r (a+b)^r = z(r-\sqrt{r}) \Rightarrow (a^r - b^r)^r \Rightarrow ((\sqrt[r]{r}-r) - (\sqrt[r]{r}+\sqrt[r]{r}))^r$$

$$(r\sqrt[r]{r} - r\sqrt[r]{r})^r = z(r-\sqrt{r})$$

$$r^r + 1 - 1\sqrt[r]{r} = z(r-\sqrt{r}) \Rightarrow r^r - 1\sqrt{r} = 14(r-\sqrt{r}) \Rightarrow z = \boxed{14}$$

$$a + \frac{1}{a} = r \quad (r+\sqrt{r})^r (r-\sqrt{r})^r \Rightarrow (r^r - r)^r \quad r - r\sqrt{r} = (r-\sqrt{r})^r$$

$$\sqrt[r]{r-\sqrt{r}} = a \quad \sqrt[r]{r+\sqrt{r}} = \frac{1}{a} \quad r = a + \frac{1}{a} = \sqrt[r]{r-\sqrt{r}} + \sqrt[r]{r+\sqrt{r}}$$

$$r^r = (r-\sqrt{r})^r + (r+\sqrt{r})^r + r(\sqrt{r-\sqrt{r}})(\sqrt{r+\sqrt{r}}) = r + r\sqrt{r} = 4$$

$$\Rightarrow (r^r - r)^r = (4-r)^r = 14 \Rightarrow r^r \rightarrow z = r$$

$$A = \sqrt[r]{r\sqrt[r]{14}} \left(\frac{1}{r}\right)^{-\frac{r}{r}} \Rightarrow \sqrt[r]{r^r \sqrt[r]{r^r}} \Rightarrow \sqrt[r]{r^{10}} \Rightarrow \sqrt[r]{r^r} \times \sqrt[r]{r^r}$$

$$A = \sqrt[r]{r^4} \quad rA = r\sqrt[r]{r^4} \Rightarrow \sqrt[r]{r^4}$$

$$(rA)^{-\frac{1}{r}} = \left(\frac{1}{rA}\right)^{\frac{1}{r}} = \left(\frac{1}{r\sqrt[r]{r^4}}\right)^{\frac{1}{r}} = \boxed{\frac{1}{r}}$$

$$(rua)^{\frac{10}{r}} = \sqrt[r]{a} \quad a^{\frac{1}{r}} = (rua)^{\frac{10}{r}} \Rightarrow a^{-r} = ru \quad a = \frac{1}{r\sqrt[r]{r}}$$

$$\frac{1}{\frac{1}{r\sqrt{r}}} - r = \frac{r\sqrt{r} - r}{1 + \sqrt{r}} = \frac{r(\sqrt{r}-1)}{1 + \sqrt{r}} \times \frac{(1-\sqrt{r})}{(1-\sqrt{r})} = \frac{r(r-\sqrt{r})}{r}$$

$$\Rightarrow 4 - r\sqrt{r}$$

$$\sqrt{x+a} = A \quad \sqrt{x-r} = B$$

$$A - B = r$$

$$A^r - B^r = (x+a) - (x-r) = r$$

$$(A-A)(B-B) = \frac{r+a}{r}$$

$$r(A+B) = \frac{r+a}{r} \Rightarrow A+B = \frac{r+a}{r}$$

$$A+B-r = \frac{r+a}{r} - r = \frac{a}{r}$$

$$\frac{a}{r}$$