

نام و نام خانوادگی: (پاسخنامه تشریحی تکلیف شماره ۲۱... کلاس ۴م دبیرستان A. سنبله ۱۳۹۸)

$$(21 + 41 + 61 + \dots + 201)(11 + 31 + 51 + \dots + 191)$$

از ۱۵ به بعد ← رقم یکان صفر می باشد چون عامل ۲ دارد.

$$11 + 31 \Rightarrow 1 + 9 = 10 \quad 21 + 41 \Rightarrow 2 + 16 = 18 \quad \Rightarrow 29 \times 7 \Rightarrow (7) \Rightarrow \text{رقم یکان}$$

۱

الف) $\sqrt{(4+\sqrt{7})^{-1}} \sqrt{(1+\sqrt{7})} = \sqrt{\frac{1}{4+\sqrt{7}}} \sqrt{(1+\sqrt{7})^2} = \sqrt{\frac{1}{4+\sqrt{7}}} \sqrt{8+2\sqrt{7}} = \sqrt{\frac{8+2\sqrt{7}}{4+\sqrt{7}}}$
 پاسخ الف: $\boxed{=\sqrt{2}}$

ب) $(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2}) \times (-\sqrt{2}) = -\frac{2+\sqrt{10}}{\sqrt{10}+2} = \boxed{-1}$ جواب نهایی ج. حاصل بستن آزمون $\sqrt{2}$ - یاس نوشته شده است.

۲

$A = \sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}} \xrightarrow{\text{تکلیف}} A^2 = 3 - \sqrt{5} + 3 + \sqrt{5} - 2\sqrt{(3-\sqrt{5})(3+\sqrt{5})} = 2$
 $A < 0 \Rightarrow A = -\sqrt{2}$

الف) $\frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{6}} - 2(\sqrt{9}-1)^{-1} = \frac{(2\sqrt{2}+3\sqrt{3})(5+\sqrt{6})}{(5-\sqrt{6})(5+\sqrt{6})} - \frac{2(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$
 $\sqrt{3^2-1} = \sqrt{3}-1$
 $\frac{10\sqrt{2}+6\sqrt{3}+15\sqrt{6}+9\sqrt{2}}{25-6} - \sqrt{3}-1 = \frac{19\sqrt{2}+19\sqrt{3}}{19} - \sqrt{3}-1 = \frac{19(\sqrt{2}+\sqrt{3})}{19} - \sqrt{3}-1$
 پاسخ الف: $\boxed{=\sqrt{2}-1}$

۳

ب) $\sqrt{3}-1+2+\sqrt{3}=2\sqrt{3}+1$
 $(2-\sqrt{3})^{-1} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{4+2\sqrt{3}}{4-3} = 4+2\sqrt{3}$
 پاسخ ب: $\boxed{1+2\sqrt{3}}$

الف) $\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} - 2 \Rightarrow A^2 = \left(\frac{\sqrt{\sqrt{3}+1}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} \right)^2 = \frac{\sqrt{3}+1+\sqrt{3}-1+2\sqrt{2}}{\sqrt{3}-\sqrt{2}}$
 $A^2 = \frac{2(\sqrt{3}+\sqrt{2})}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{2(\sqrt{3}+\sqrt{2})^2}{1} \Rightarrow A = \sqrt{2}(\sqrt{3}+\sqrt{2}) = \sqrt{6}+2$
 ب) $\sqrt[4]{3^8} \times \sqrt[4]{192} \times \sqrt[4]{4\sqrt{2}} = (2^2)^{\frac{1}{4}} \times \sqrt[4]{2 \times 3^5} \times 2^{\frac{1}{4}} \sqrt[4]{2} = 2^{\frac{2}{4}} \times 3^{\frac{5}{4}} \times 2^{\frac{1}{4}} \times 2^{\frac{1}{4}} = 2 \times 3^{\frac{5}{4}} = 2 \times 3^1 \times 3^{\frac{1}{4}} = 12\sqrt[4]{3}$

۴

$3^x + 3^{x+1} + 3^{x+2} + 3^{x+3} + 3^{x+4} + 3^{x+5}$
 $3^x(1+3+3^2+3^3+3^4+3^5) \Rightarrow \frac{3^6-1}{3-1} = \frac{728}{2} = 364$
 صورت $364 \cdot 3^x$
 $3^{x-2} + 3^{x-1} + 3^x + 3^{x+1} + 3^{x+2} + 3^{x+3}$
 $\Rightarrow 3^{x-2}(1+3+3^2+3^3+3^4+3^5)$
 $= \frac{3^6-1}{3-1} = 364$
 $\frac{364 \cdot 3^x}{4 \cdot 3^{x-2}} = 52 \Rightarrow \frac{364}{4} = 52 \Rightarrow \frac{52}{9} \times \frac{3^x}{3^{x-2}} = 52$
 $\frac{1}{9} \times \frac{3^x}{3^{x-2}} = 1 \Rightarrow \frac{3^x}{3^{x-2}} = 9 \Rightarrow 3^2 = 9 \Rightarrow \boxed{x=2}$

۵

$$(a-b)^r (a+b)^r \Rightarrow (a^r - b^r)^r$$

$$(\sqrt{\sqrt{4}+2} - \sqrt{\sqrt{4}-2})^r = (\sqrt{4} + 2 + \sqrt{4} - 2 - 2\sqrt{2})^r = (2\sqrt{4} - 2\sqrt{2})^r$$

$$= 2(4+2-2\sqrt{2}) = 12(2-\sqrt{2}) = \frac{14}{2}(2-\sqrt{2})$$

$$\boxed{t=14}$$

6

$$((a + \frac{1}{a})^r - 2)^r = (a^r + \frac{1}{a^r} + \cancel{2} - 2)^r = a^r + \frac{1}{a^r} + 2$$

$$2 - 2\sqrt{2} + \frac{1}{2-2\sqrt{2}} + 2 = 9 - 2\sqrt{2} + \frac{2+2\sqrt{2}}{4-4} = 2^r = 14$$

$$\boxed{t=2}$$

7

$$A = \sqrt[r]{r^r \sqrt[r]{r^r}} \times r^{\frac{r}{r}} = \sqrt[r]{r^r \times r^{\frac{r}{r}}} \times r^{\frac{r}{r}} = \sqrt[r]{r^{\frac{10}{r}}} \times r^{\frac{r}{r}} \Rightarrow A = (r^{\frac{10}{r}})^{\frac{1}{r}} \times r^{\frac{r}{r}}$$

$$= r^{\frac{r}{r}} \times r^{\frac{r}{r}} = r^r \Rightarrow (rA)^{-\frac{1}{r}} = (r^r)^{-\frac{1}{r}} = r^{-1} = \boxed{0.5}$$

8

$$a^{\frac{1}{2}} = 2\sqrt{a} \xrightarrow{\div a^{\frac{1}{2}}} a^{-\frac{1}{2}} = 2\sqrt{a} \Rightarrow a^{-r} = 2\sqrt{a} \Rightarrow a^r = \frac{1}{2\sqrt{a}}$$

$$a > 0 \Rightarrow a = \frac{1}{\sqrt{2\sqrt{a}}} = \frac{1}{2\sqrt{a}} \quad \frac{1}{a} = 2\sqrt{a} \Rightarrow \frac{1}{a} - 2 = 2\sqrt{a} - 2$$

$$= 2(\sqrt{a} - 1) \Rightarrow 2(\sqrt{a} - 1) = K(1 + \sqrt{a}) \Rightarrow K = 2 \frac{\sqrt{a} - 1}{1 + \sqrt{a}}$$

$$\frac{\sqrt{a} - 1}{1 + \sqrt{a}} = 2 - \sqrt{a} \Rightarrow K = 2(2 - \sqrt{a}) \Rightarrow (\frac{1}{a} - 2) \rightsquigarrow \boxed{2(2 - \sqrt{a})}$$

9

$$u = \sqrt{x+a}, v = \sqrt{x-f} \Rightarrow u-v = 2$$

$$u^r - v^r = (x+a) - (x-f) = a+f$$

$$u^r - v^r = (u-v)(u+v) \Rightarrow 2(u+v) = a+f \Rightarrow u+v = \frac{a+f}{2}$$

$$\sqrt{x+a} + \sqrt{x-f} - 2 = (u+v) - 2 \Rightarrow \frac{a+f}{2} - 2 = \boxed{\frac{a-f}{2}}$$

10