

(1)

$$(1! + 3! + 5! + \dots + 25!) (2! + 4! + 6! + \dots + 24!)$$

$$\downarrow$$

$$1! + 3! = 7$$

$$1! + 3! + 5! = 127$$

به دلیل عوامل ۲ در ۱ و ۳ و ۵
در بیان می‌نویسند

$$\text{بیان} = 7$$

$$\downarrow$$

$$2! + 4! = 24$$

$$2! + 4! + 6! = 744$$

به دلیل عوامل ۲ در ۲ و ۴ و ۶
در بیان می‌نویسند

$$\text{بیان} = 4$$

$$4 \times 7 = 28$$

بیان ۲۸

(2)

الف) $\sqrt[4]{(2+\sqrt{5})^{-1}} \times \sqrt{1+\sqrt{5}}$

$$= \sqrt[4]{\frac{1}{2+\sqrt{5}}} \times \sqrt[4]{1+\sqrt{5}}$$

$$= \sqrt[4]{2}$$

$$= \sqrt[4]{\frac{1+\sqrt{5}}{2+\sqrt{5}}} = \sqrt[4]{\frac{2(2+\sqrt{5})}{(2+\sqrt{5})}}$$

ب) $\left(\frac{\sqrt{2} + \sqrt{5}}{\sqrt{5} + 2} \right) (\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}}) = \frac{(\sqrt{2} + \sqrt{5})}{(\sqrt{2})(\sqrt{5} + \sqrt{3})} (\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}})$

$$\rightarrow = \frac{(\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}})(\sqrt{3-\sqrt{5}} + \sqrt{3+\sqrt{5}})}{(\sqrt{2})(\sqrt{5} + \sqrt{3})} = \frac{3-\sqrt{5} - 3-\sqrt{5}}{(\sqrt{2})(\sqrt{5} + \sqrt{3})} = \frac{-2\sqrt{5}}{(\sqrt{2})(\sqrt{5} + \sqrt{3})} = -1$$

$$\begin{aligned}
 \text{الف) } \frac{\sqrt{1} + \sqrt{16}}{8 - \sqrt{4}} - 2(\sqrt{9} - 1)^{-1} &= \frac{2\sqrt{4} + 4\sqrt{4}}{8 - \sqrt{4}} - \frac{2}{\sqrt{9} - 1} \\
 &= \frac{(2\sqrt{4} + 4\sqrt{4})}{(8 - \sqrt{4})} \times \frac{(\sqrt{4} + \sqrt{4})}{(\sqrt{4} + \sqrt{4})} - \frac{(2)}{(\sqrt{9} - 1)} \times \frac{(\sqrt{9} + 1)}{(\sqrt{9} + 1)} \\
 &= \frac{(2\sqrt{4} + 4\sqrt{4})(\sqrt{4} + \sqrt{4})}{(8 - \sqrt{4})(\sqrt{4} + \sqrt{4})} - \frac{2(\sqrt{9} + 1)}{9 - 1} = 2 + \sqrt{4} - \sqrt{9} - 1 \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \text{ب) } \frac{\sqrt{16} - 1}{9 + \sqrt{9}} + (2 - \sqrt{9})^{-1} &= \frac{(4\sqrt{9} - 1)}{(9 + \sqrt{9})} \times \frac{(9 - \sqrt{9})}{(9 - \sqrt{9})} + \frac{(1)}{(2 - \sqrt{9})} \times \frac{(2 + \sqrt{9})}{(2 + \sqrt{9})} \\
 &= \frac{12\sqrt{9} - 9 - 9 + \sqrt{9}}{18} + \frac{(2 + \sqrt{9})}{1} = \frac{13(\sqrt{9} - 1)}{18} + 2 + \sqrt{9} = 2\sqrt{9} + 1
 \end{aligned}$$

$$\begin{aligned}
 \text{الف) } \frac{\sqrt{1 + \sqrt{4}} + \sqrt{\sqrt{9} - 1}}{\sqrt{\sqrt{9} - \sqrt{4}}} - 2 \sqrt{\frac{(\sqrt{\sqrt{9} + 1} + \sqrt{\sqrt{9} - 1})^2}{(\sqrt{\sqrt{9} - \sqrt{4}})^2}} - 2 \\
 = \sqrt{\frac{\sqrt{9} + 1 + \sqrt{9} - 1 + 2\sqrt{4}}{\sqrt{9} - \sqrt{4}}} - 2 = \sqrt{\frac{(2\sqrt{9} + 2\sqrt{4})(\sqrt{9}\sqrt{4})}{(\sqrt{9} - \sqrt{4})(\sqrt{9} + \sqrt{4})}} - 2 \\
 = \sqrt{\frac{2(\sqrt{9} + \sqrt{4})^2}{9 - 4}} - 2 = \sqrt{4 + 2 - 2} = \sqrt{4}
 \end{aligned}$$

$$b) \sqrt[10]{\sqrt[10]{16}} \times \sqrt[10]{144} \times \sqrt[10]{\frac{4}{9}}$$

$$= \sqrt[10]{16} \times \sqrt[10]{144} \times \sqrt[10]{\frac{4}{9}} = \sqrt[10]{16 \times 144 \times \frac{4}{9}}$$

$$= \sqrt[10]{16 \times 144 \times \frac{4}{9}} = 16 \times 12 \times 2 = 384$$

$$\frac{r^2 + r^{2+1} + r^{2+2} + r^{2+3} + r^{2+4} + r^{2+5}}{r^{2-2} + r^{2-1} + r^{2-0} + r^{2+1} + r^{2+2} + r^{2+3}}$$

$$= \omega r$$

$$\rightarrow \frac{r^2 (1 + r + r^2 + r^3 + r^4 + r^5)}{r^2 \left(\frac{1}{r} + 1 + r + r^2 + r^3 + r^4 \right)} = \frac{r^2 \times 346}{r^2 \times \frac{4r}{r}} = \omega r$$

$$\rightarrow \left(\frac{r}{r} \right)^2 = \frac{346}{4r} = \frac{9}{r} \rightarrow r = 9$$

$$a = \sqrt[10]{\sqrt[10]{4-2}}, b = \sqrt[10]{\sqrt[10]{4+2}}$$

$$\begin{aligned} & (a^2 + b^2 - 2ab)(a^2 + b^2 + 2ab) = \pm (2 - \sqrt{2}) \\ & = (a^4 + b^4 + 2a^2b^2 - 2a^2b^2) = (\sqrt{4-2} + \sqrt{4+2} - 2\sqrt{2})^2 = (2\sqrt{2}(\sqrt{2}-1))^2 \\ & = 16(2-2\sqrt{2}) = 16(2-\sqrt{2}) = \pm (2-\sqrt{2}) \rightarrow \pm = 16 \end{aligned}$$

$$\sqrt{x+a} - \sqrt{x+b} = y \longrightarrow x+a - x+b = y(\sqrt{x+a} + \sqrt{x+b})$$

$$\Rightarrow \frac{a+b}{y} = \sqrt{x+a} + \sqrt{x+b}$$

$$\sqrt{x+a} + \sqrt{x+b} - y = \frac{a+b}{y} - y = \frac{a}{y}$$