۴) الف) ۱٫۵

$$\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}\times\frac{\sqrt{\sqrt3+\sqrt2}}{\sqrt{\sqrt3+\sqrt2}}\Rightarrow\frac{\left(\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}\right)\times\sqrt{\sqrt3+\sqrt2}}{\sqrt{3-2=1}=1}$$

$$\left(\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}\right)\left(\sqrt{\sqrt3+\sqrt2}\right)\to 3+\sqrt2+\sqrt3+\sqrt6+3+\sqrt6-\sqrt3-\sqrt2=6+2\sqrt6$$

$$6-2\sqrt6-2=\cancel{4-2\sqrt6}$$

ب)

$$2^{\frac{8}{12}}\times3\times2^{\frac14}\times\sqrt[12]{2^{13}}\to2^{\frac{8}{12}+\frac{3}{12}+\frac{13}{12}=\frac{24}{12}}\times3\to2^2\times3=12$$

۵) ۲

$$\frac{3^x\left(1+3+3^2+3^3+3^4+3^5\right)}{2^{x-2}\left(1+2+2^2+2^3+2^4+2^5\right)}\to\frac{3^x\left(1\left(\frac{3^6-1}{2}\right)\right)}{2^{x-2}\left(1\left(\frac{2^6-1}{1}\right)\right)}\Rightarrow\frac{3^x(364)}{2^{x-2}(63)}=1\to$$

$$\frac{3^x\times7\times1}{2^{x-2}\times63\times9}=\frac{3^x\times3^{-2}}{2^{x-2}}\to\frac{3^{x-2}}{2^{x-2}}=1$$

$$3^{x-2}=2^{x-2}\qquad x-2=0\to\boxed{x=2}$$

- باید توان ها برابر باشد.

۶)

$$(a+b)^2(a-b)^2-(a+b)(a-b)(a+b)(a-b)\to(a^2-b^2)^2\to a^4+b^4-2a^2b^2$$

۱)

$$\sqrt6+\cancel{2}+\sqrt6-\cancel{2}\;.\;-2\left(\sqrt{\sqrt{6^2}-4}\right)\to2\sqrt6-2\sqrt2\to2\left(\sqrt6-\sqrt2\right)$$

$$\frac{2(\sqrt6-\sqrt2)}{1(2-\sqrt3)}\to\frac{2(\sqrt6-\sqrt2)}{2-\sqrt3}\to\frac{2(\sqrt6-\sqrt2)(2+\sqrt3)}{1}=+=\cancel{\sqrt6+\sqrt2}$$

۷) ۲

$$\left(\left(a+\frac1a\right)^2-2\right)^2\to\left(a^2+\frac1{a^2}\right)^2+\cancel{4}-\cancel{4}\to a^4+\frac1{a^4}+2=?$$

$$a=\sqrt{7-2\sqrt{12}}\to\left(\sqrt4-\sqrt3\right)^2=\left(2-\sqrt3\right)\;\Big|\;(2-\sqrt3)^4+\frac{1}{(2-\sqrt3)^4}+2=?$$

$$\frac{(2+\sqrt3)^4}{(2+\sqrt3)^4(2-\sqrt3)^4}$$

$$7-4\sqrt3+7+4\sqrt3=14\to14+2=16\to?=\boxed{4}$$

۱۹۶

آروین عباسی

۱۸٫۲۵

۱) ۲

$1! \to 1$ $\quad 4! \to 24$

$2! \to 2$ $\quad 5! \to 120$

$3! \to 6$ $\quad 6! \to 720$

$(1! + 3! + 5! + 7! + \dots)$ اعداد فرد

$\frac{6+120+1}{7}$

$6 \times 7 = 42$

$\frac{3!+4!+6!}{7^6}$

$\frac{24+2}{24}$

$2 =$ احتمال

۲) ۲

الف) $\sqrt[4]{\frac{1}{4+\sqrt{7}}} \times \sqrt[4]{(1+\sqrt{7})^2} \Rightarrow \sqrt[4]{\frac{1+7+2\sqrt{7}}{4+\sqrt{7}}} \to \sqrt[4]{\frac{8+2\sqrt{7}}{4+\sqrt{7}}} = \sqrt[4]{2}$

ب) $\frac{\sqrt{2}+\sqrt{5}\,(\sqrt{10}-2)}{10-4=6} \Rightarrow \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2} \Rightarrow \left(\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}}\right)\frac{\sqrt{2}}{2} \to \sqrt{6-2\sqrt{5}} - \sqrt{6+2\sqrt{5}}$

$\to \sqrt{6+2\sqrt{5}} \to \sqrt{(\sqrt{5}+1)^2} \to \cancel{\sqrt{5}-1}\ \frac{\sqrt{5}-1-\sqrt{5}-1}{2} \to \frac{-2}{2} = -1$

۳) ۲

الف) $\frac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}} - 2(\sqrt{3}-1)^{-1} \to \frac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}} - \frac{2}{\sqrt{3}-1}$

$\frac{(2\sqrt{2}+3\sqrt{3})(5+\sqrt{6})}{\underbrace{(5-\sqrt{6})(5+\sqrt{6})}_{25-6=19}} \Rightarrow \frac{19\sqrt{3}+19\sqrt{2}}{19} = \sqrt{3}+\sqrt{2}$

$\frac{2(\sqrt{3}+1)}{3-1} = \sqrt{3}+1$

$\Rightarrow \sqrt{3}+\sqrt{2}-\sqrt{3}-1 \Rightarrow \sqrt{2}-1$

ب) $\frac{3\sqrt{3}-1}{4+\sqrt{3}} + \frac{1}{2-\sqrt{3}} = \sqrt{3}-1+2+\sqrt{3} \Rightarrow 2\sqrt{3}+1$

$\frac{-1+3\sqrt{3}(4-\sqrt{3})}{13} \Rightarrow \frac{12\sqrt{3}-9-4+\sqrt{3}}{13} = \frac{13\sqrt{3}-13}{13} = \sqrt{3}-1$

$\frac{2+\sqrt{3}}{4-3=1} \Rightarrow 2+\sqrt{3}$

۸)

$A=\sqrt[5]{\sqrt[3]{2^{10}}}=2^{\frac{10}{15}}=2^{\frac{2}{3}}\times 2^{\frac{4}{3}}\to 2^{\frac{6}{3}}\to 2^{2}=4=A$ ✓

۲

$\sqrt[3]{-\frac{1}{2\times 4}-\frac{1}{8}}\to\sqrt[3]{-\lambda}=\left(\frac{1}{2}\right)$ ✓

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۹)

$27a^{\frac{15}{27}}=\sqrt[7]{a}\to a\to 27\left(3^{3}\times a^{\frac{15}{7}}\right)^{7}\Rightarrow 3^{21}\times a^{15}=2$

۱٫۷۵

$a^{-4}=3^{21}\to a^{-3}=\frac{3^{3}}{27}-\cancel{\ldots}\ \underbrace{a^{2}=(3^{3})^{2}}\to\left(\frac{1}{a}-3\right)=\cancel{\ldots}$

$\frac{1}{3\sqrt{3}}=\frac{\sqrt{3}}{9}$ $\qquad \alpha=\frac{1}{\sqrt{27}}\to a=\frac{1}{3\sqrt{3}}$

$\frac{1}{a}=\frac{1}{\frac{\sqrt{3}}{9}}=\frac{9}{\sqrt{3}}=\frac{9\sqrt{3}}{3}=3\sqrt{3}$ $\qquad 3\sqrt{3}-3\to 3(\sqrt{3}-1)\to\frac{3(\sqrt{3}-1)}{1+\sqrt{3}}$

$\cancel{3(1-\sqrt{3})}=\frac{3\times 2(1-\sqrt{3})}{2}$ $\quad =\frac{3(1+\cancel{3-2\sqrt{3}})}{2}$ ($4-2\sqrt{3}$) درست! $\leftarrow\frac{3(\sqrt{3}-1)(\sqrt{3}-1)}{3-1=2}$

$\boxed{6-3\sqrt{3}}$

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۱۰)

$\sqrt{x+a}=\sqrt{x-4}+2\Rightarrow x+a=(x-4)+4\sqrt{x-4}+4\to$

۲

$x+a=x+4(\sqrt{x-4})\to a=4\sqrt{x-4}\to\frac{a}{4}=\sqrt{x-4}\to x-4=\frac{a^{2}}{16}$

$x=\frac{a^{2}}{16}+4$

$\cancel{\sqrt{x-a}+\frac{a}{4}-2}\Rightarrow x-a=\frac{a^{2}}{16}+4-a\to\sqrt{x-a}=\sqrt{\frac{a^{2}}{16}-a+4}\to$

$\sqrt{\frac{a^{2}-16a+64}{16}}\Rightarrow\sqrt{\frac{(a-8)^{2}}{16}}=\frac{|a-8|}{4}$

$\sqrt{x+a}+\sqrt{x-4}-2=0\to\sqrt{x-4}+2+\sqrt{x-4}-2=2\sqrt{x-4}$

$\sqrt{x+a}=\sqrt{x-4}+2$ $\qquad \sqrt{x+a}+\sqrt{x-4}-2=2\sqrt{x-4}=2\times\frac{a}{4}=\left(\frac{a}{2}\right)$ ✓

۴_ الف)

$$\frac{\overbrace{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}^{A}}{\sqrt{\sqrt{3}-\sqrt{2}}}-2=\frac{\sqrt{2}\left(\sqrt{\sqrt{3}+\sqrt{2}}\right)}{\sqrt{\sqrt{3}-\sqrt{2}}}-2=\sqrt{2}\left(\sqrt{\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}}\right)-2=$$

$$A^2=2\sqrt{3}+2\sqrt{2}\rightarrow A=\sqrt{2}\left(\sqrt{\sqrt{3}+\sqrt{2}}\right)$$

$$\sqrt{2}\sqrt{(\sqrt{3}+\sqrt{2})^2}=\sqrt{6}+2-2=\boxed{\sqrt{6}}$$

۶_

$$((a-b)^2)^2((a+b)^2)^2=(a^2-b^2)^4=\left(\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2}\right)^4=$$

$$\left(\sqrt{6}-2+\sqrt{6}+2-2\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)}\right)^2=(2\sqrt{6}-2\sqrt{2})^2=16(2-\sqrt{3})$$