۱)
$(16+26+51+\dots+256)(21+41+61+\dots+246) = 6\times7=42 \to$ رقم یکان $=2$

7 ، $24+2=26 \to 6$

۲) الف) $\frac{1}{\sqrt[4]{4+\sqrt7}}\times\sqrt[4]{1+\sqrt7} \longrightarrow \frac{\sqrt[4]{(4+\sqrt7)^3}}{4+\sqrt7}\times\sqrt[4]{(1+\sqrt7)^2} \Rightarrow (4+\sqrt7)^{-\frac14}\times((1+\sqrt7)^2)^{\frac14}$

$\longrightarrow \left(\frac{1}{4+\sqrt7}\right)^{\frac14}\times((1+\sqrt7)^2)^{\frac14} = \left(\frac{(1+\sqrt7)^2}{4+\sqrt7}\right)^{\frac14} = \left(\frac{8+2\sqrt7}{4+\sqrt7}\right)^{\frac14} = 2^{\frac14} = \sqrt[4]{2}$

ب) $\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\times\frac{\sqrt{10}-2}{\sqrt{10}-2} = \frac{\sqrt2}{2} \to \left(\frac{\sqrt2}{2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) \to \frac{\sqrt{6-2\sqrt5}-\sqrt{6+2\sqrt5}}{2}$

$\frac{\sqrt{(\sqrt5-1)^2}-\sqrt{(\sqrt5+1)^2}}{2} \Rightarrow \frac{\sqrt5-1-\sqrt5-1}{2} = \frac{-2}{2} = -1$

۳) الف) $\frac{2\sqrt2+3\sqrt3}{5-\sqrt6} - \frac{2}{\sqrt3-1}$ → گویا سازی کسر اولی $\Rightarrow \frac{19(\sqrt3+\sqrt2)}{19} = \sqrt3+\sqrt2$

گویا سازی کسر دوم: $\frac{2(\sqrt3+1)}{3-1} = \sqrt3+1 \Rightarrow \sqrt3+\sqrt2-\sqrt3-1 = \sqrt2-1$

ب) $\frac{3\sqrt3-1}{4+\sqrt3}+\frac{1}{2-\sqrt3} \Rightarrow$ گویا سازی کسر اول $= \frac{3\sqrt3-1}{4+\sqrt3}\times\frac{4-\sqrt3}{4-\sqrt3} = \frac{13(\sqrt3-1)}{13} = \sqrt3-1$

گویا سازی دوم $= \frac{2+\sqrt3}{4-3} = 2+\sqrt3 \to \sqrt3-1+2+\sqrt3 = 2\sqrt3+1$

۴) الف) $\frac{\sqrt{1+\sqrt3+\sqrt{\sqrt3-1}}}{\sqrt{\sqrt3-\sqrt2}}\times\frac{\sqrt{\sqrt3+\sqrt2}}{\sqrt{\sqrt3+\sqrt2}} \to \frac{3+\sqrt2+\sqrt3+\sqrt6+3+\sqrt6-\sqrt3-\sqrt2}{1} = 6+2\sqrt6$

$\longrightarrow 6+2\sqrt6-2 =$ ~~$4+2\sqrt6$~~

ب) $(2^{\frac83})^{\frac12}\times3(2)^{\frac14}\times((2^{\frac16})\times2^2)^{\frac12} \Rightarrow 2^{\frac43}\times3(2)^{\frac14}\times(2)^{\frac{13}{12}} \Rightarrow 2^{\frac{24}{12}}\times3 = 4\times3 = 12$

۴- الف)

$$\overbrace{\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}}^{A}-2=\frac{\sqrt{2}\left(\sqrt{\sqrt{3}+\sqrt{2}}\right)}{\sqrt{\sqrt{3}-\sqrt{2}}}-2=\sqrt{2}\left(\sqrt{\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}}\right)-2=$$

$$A^2=2\sqrt{3}+2\sqrt{2}\rightarrow A=\sqrt{2}\left(\sqrt{\sqrt{3}+\sqrt{2}}\right)$$

$$\sqrt{2}\sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2}=\sqrt{6}+2-2=\boxed{\sqrt{6}}$$