

$(16 + 36 + 64 + \dots + 256) (216 + 216 + 216 + \dots + 216) = 9 \times 42 \rightarrow 9 \times 42 = 378$
 $(16 + 36 + 64 + \dots + 256) = 256 \times 5 = 1280$
 $216 + 216 + 216 + \dots + 216 = 216 \times 5 = 1080$
 $1280 \times 1080 = 1382400$
 $1382400 \div 378 = 3657.14$

الف) $\frac{1}{\sqrt[4]{4+\sqrt{2}}} \times \sqrt{1+\sqrt{2}} \rightarrow \frac{\sqrt[4]{(4+\sqrt{2})^2}}{4+\sqrt{2}} \times \sqrt[4]{(1+\sqrt{2})^2} = (4+\sqrt{2})^{-\frac{1}{2}} \times ((1+\sqrt{2})^2)^{\frac{1}{4}}$

$\Rightarrow \left(\frac{1}{4+\sqrt{2}}\right)^{\frac{1}{2}} \times ((1+\sqrt{2})^2)^{\frac{1}{4}} = \left(\frac{(1+\sqrt{2})^2}{4+\sqrt{2}}\right)^{\frac{1}{4}} = \left(\frac{1+2\sqrt{2}}{4+\sqrt{2}}\right)^{\frac{1}{4}} = 2^{\frac{1}{4}} = \sqrt[4]{2}$

ب) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2} \times \frac{\sqrt{10}-2}{\sqrt{10}-2} = \frac{\sqrt{2}}{2} \rightarrow \left(\frac{\sqrt{2}}{2}\right)(\sqrt{10}-2) \rightarrow \frac{\sqrt{20}-2\sqrt{2}}{2}$

$\frac{\sqrt{(\sqrt{5}-1)^2} - \sqrt{(\sqrt{5}+1)^2}}{2} = \frac{\sqrt{5}-1 - \sqrt{5}-1}{2} = \frac{-2}{2} = -1$

الف) $\frac{2\sqrt{2}+3\sqrt{2}}{5-\sqrt{2}} - \frac{2}{\sqrt{2}-1} \rightarrow \frac{19(\sqrt{2}+1)}{19} = \sqrt{2}+1$

ب) $\frac{2(\sqrt{2}+1)}{3-1} = \sqrt{2}+1 \Rightarrow \sqrt{2}+1 - \sqrt{2}-1 = 0$

ب) $\frac{3\sqrt{2}-1}{4+\sqrt{2}} + \frac{1}{2-\sqrt{2}} \Rightarrow \frac{3\sqrt{2}-1}{4+\sqrt{2}} \times \frac{4-\sqrt{2}}{4-\sqrt{2}} = \frac{12(\sqrt{2}-1)}{14} = \sqrt{2}-1$

ب) $\frac{2+\sqrt{2}}{4-2} = 1+\sqrt{2} \rightarrow \sqrt{2}-1 + 1+\sqrt{2} = 2\sqrt{2}$

الف) $\frac{\sqrt{1+\sqrt{2}}+\sqrt{1-\sqrt{2}}}{\sqrt{2}-\sqrt{2}} \times \frac{\sqrt{2}+\sqrt{2}}{\sqrt{2}+\sqrt{2}} \rightarrow \frac{3+\sqrt{2}+\sqrt{2}+\sqrt{2}+3+\sqrt{2}-\sqrt{2}-\sqrt{2}}{1} = 9+2\sqrt{2}$

$\rightarrow 9+2\sqrt{2} - 2\sqrt{2} = 9$

ب) $(\sqrt[4]{2})^{\frac{1}{2}} \times 3(\sqrt[4]{2})^{\frac{1}{2}} \times \left(\left(\sqrt[4]{2}\right)^{\frac{1}{2}} \times \sqrt[4]{2}\right)^{\frac{1}{2}} = 2^{\frac{1}{4}} \times 3 \times 2^{\frac{1}{4}} \times 2^{\frac{1}{4}} = 2^{\frac{3}{4}} \times 3 = 4\sqrt{2} = 12$

$$\frac{r^n (1 + r + r^2 + \dots + r^{n-1})}{r^n (\frac{1}{2} + \frac{1}{r} + 1 + r + \dots + r^{n-1})} = \left(\frac{r}{r}\right)^n \left(\frac{1 \left(\frac{r^n - 1}{r - 1}\right)}{\frac{1}{2} \left(\frac{r^n - 1}{r - 1}\right)}\right) = \left(\frac{r}{r}\right)^n \left(\frac{r^2}{10/10}\right) = 10 \quad (1)$$

$$\rightarrow \left(\frac{r}{r}\right)^n = 10 \rightarrow \boxed{n=2} \quad (2)$$

$$\rightarrow (a^r - b^r)^r \rightarrow \left(\left(\sqrt{9-5} - \sqrt{5+5}\right)^r\right)^r \Rightarrow \sqrt{9-5} + \sqrt{5+5} - 2\sqrt{5} = 2\sqrt{5} - 2\sqrt{5} \rightarrow \sqrt{9-5}$$

$$\left((2)(\sqrt{9-5})\right)^r \rightarrow (4)(9+5-4\sqrt{5}) = 4(14-4\sqrt{5}) \Rightarrow (4)(14-4\sqrt{5}) \rightarrow 19(2-\sqrt{5})$$

$$\rightarrow \boxed{t=19} \quad \checkmark$$

$$\sqrt[r]{r-2\sqrt{5}} \rightarrow \sqrt[r]{(\sqrt{9-5}-\sqrt{5+5})^r} = \sqrt[r]{r-2\sqrt{5}} = a \quad (3)$$

$$\left(\left(a + \frac{1}{a} + \sqrt{5}\right)\left(a + \frac{1}{a} - \sqrt{5}\right)\right)^r \rightarrow \left(a^r + \frac{1}{a^r} + r - r\right)^r = a^r + \frac{1}{a^r} + r = r^t \quad (4)$$

$$2a^r = (r-2\sqrt{5})^r + \frac{1}{(r-2\sqrt{5})^r} + r \Rightarrow r-2\sqrt{5} + \frac{1}{r-2\sqrt{5}} + r \rightarrow r-2\sqrt{5} + r+2\sqrt{5} + r = 19$$

$$19 = r^r = r^t \rightarrow \boxed{t=r}$$

$$A = \left(r^r \times r^{\frac{r}{r}}\right)^{\frac{1}{r}} \times r^{\frac{r}{r}} \rightarrow \left(r^{\frac{1}{r}}\right)^{\frac{1}{r}} = r^{\frac{1}{r^2}} \rightarrow r^{\frac{r}{r^2}} \times r^{\frac{r}{r^2}} = r^{\frac{r}{r}} = r = A \quad (5)$$

$$(r \times r)^{-\frac{1}{r}} = r^{-\frac{1}{r}} = \left(\frac{1}{r}\right)^{\frac{1}{r}} = \frac{1}{r^{\frac{1}{r}}} = \boxed{\frac{1}{r}} \quad \checkmark$$

$$\sqrt[r]{a} = r \times a^{\frac{1}{r}} \rightarrow a = \left(r^r \times a^{\frac{1}{r}}\right)^r \rightarrow a = r^r \times a^{\frac{1}{r}} \rightarrow \frac{1}{a^{\frac{1}{r}}} = r^r \rightarrow \frac{1}{a^r} = r^r \quad (6)$$

$$a^r = \frac{1}{r^r} = a = \frac{1}{r^{\sqrt{r}}} \rightarrow \left(\frac{1}{a} - r\right) = r^{\sqrt{r}} - r = r(\sqrt{r} - 1) \rightarrow \frac{(r)(\sqrt{r} - 1)}{1 + \sqrt{r}} \times \frac{1 - \sqrt{r}}{1 - \sqrt{r}}$$

$$\frac{(r)(-1)(\sqrt{r} + 1 - \sqrt{r})}{-r} = \frac{(r)(\sqrt{r} - 1)}{-r} = \frac{(r)(r)(\sqrt{r} - 1)}{-r} = -(r)(\sqrt{r} - 1) = -r\sqrt{r} + r \quad (7)$$

$$\sqrt{m+a} = p \rightarrow p^r = m+a \rightarrow p^r - q^r = (p+q)(p^{r-1} - p^{r-2}q + \dots - q^{r-1}) = m+a - m+f$$

$$r(p+q) = r+a \rightarrow p+q = \frac{r+a}{r} \rightarrow \sqrt{m+a} + \sqrt{m-f} - r = p+q - r = \frac{r+a}{r} - r = \frac{a+f-f}{r} = \frac{a}{r} \quad \checkmark$$

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$$\overbrace{\frac{\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}}^A - 2 = \frac{\sqrt{2}(\sqrt{\sqrt{3}+\sqrt{2}})}{\sqrt{\sqrt{3}-\sqrt{2}}} - 2 = \sqrt{2} \left(\sqrt{\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}} \right) - 2 =$$

$$\rightarrow A^2 = 2\sqrt{3} + 2\sqrt{2} \rightarrow A = \sqrt{2}(\sqrt{\sqrt{3}+\sqrt{2}})$$

$$\sqrt{2} \sqrt{(\sqrt{3}+\sqrt{2})^2} = \sqrt{4} + 2 - 2 = \boxed{\sqrt{4}}$$