

$(16 + 36 + 64 + \dots + 256) (216 + 216 + 216 + \dots + 216) = 9 \times 12 = 108 \rightarrow \text{جواب} = 108$
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الف) $\frac{1}{\sqrt[4]{4+2\sqrt{2}}} \times \sqrt{1+\sqrt{2}} \rightarrow \frac{\sqrt{(4+2\sqrt{2})^2}}{4+2\sqrt{2}} \times \sqrt{(1+\sqrt{2})^2} = (4+2\sqrt{2})^{-\frac{1}{2}} \times ((1+\sqrt{2})^2)^{\frac{1}{4}}$

$\Rightarrow \left(\frac{1}{4+2\sqrt{2}}\right)^{\frac{1}{2}} \times ((1+\sqrt{2})^2)^{\frac{1}{4}} = \left(\frac{(1+\sqrt{2})^2}{4+2\sqrt{2}}\right)^{\frac{1}{2}} = \left(\frac{1+2\sqrt{2}}{4+2\sqrt{2}}\right)^{\frac{1}{2}} = 2^{\frac{1}{4}} = \sqrt[4]{2}$

ب) $\frac{\sqrt{2+2\sqrt{2}}}{\sqrt{10+2}} \times \frac{\sqrt{10-2}}{\sqrt{10-2}} = \frac{\sqrt{2}}{2} \rightarrow \left(\frac{\sqrt{2}}{2}\right)(\sqrt{10-2} - \sqrt{10+2}) \rightarrow \frac{\sqrt{10-2} - \sqrt{10+2}}{2}$

$\frac{\sqrt{(10-2)} - \sqrt{(10+2)}}{2} = \frac{\sqrt{8} - \sqrt{12}}{2} = \frac{2\sqrt{2} - 2\sqrt{3}}{2} = \sqrt{2} - \sqrt{3} = -1$

الف) $\frac{2\sqrt{2} + 3\sqrt{2}}{5 - \sqrt{2}} - \frac{2}{\sqrt{3} - 1} \rightarrow \text{گوشه سازی کسر} = \frac{19(\sqrt{3} + \sqrt{2})}{19} = \sqrt{3} + \sqrt{2}$

$\rightarrow \text{گوشه سازی کسر} : \frac{2(\sqrt{3} + 1)}{3 - 1} = \sqrt{3} + 1 \Rightarrow \sqrt{3} + \sqrt{2} - \sqrt{3} - 1 = \sqrt{2} - 1$

ب) $\frac{3\sqrt{3} - 1}{4 + \sqrt{2}} + \frac{1}{2 - \sqrt{2}} \Rightarrow \text{گوشه سازی کسر} = \frac{3\sqrt{3} - 1}{4 + \sqrt{2}} \times \frac{4 - \sqrt{2}}{4 - \sqrt{2}} = \frac{12(\sqrt{3} - 1)}{12} = \sqrt{3} - 1$

$\rightarrow \text{گوشه سازی کسر} = \frac{2 + \sqrt{2}}{4 - 2} = 1 + \sqrt{2} \rightarrow \sqrt{3} - 1 + 1 + \sqrt{2} = \sqrt{3} + \sqrt{2} + 1$

الف) $\frac{\sqrt{1+\sqrt{2}} + \sqrt{1-\sqrt{2}}}{\sqrt{10-2}} \times \frac{\sqrt{10+2}}{\sqrt{10+2}} \rightarrow \frac{3 + \sqrt{2} + \sqrt{2} + \sqrt{2} + 3 + \sqrt{2} - \sqrt{2} - \sqrt{2}}{1} = 9 + 2\sqrt{2}$

$\rightarrow 9 + 2\sqrt{2} - 2(4 + 2\sqrt{2})$

ب) $(2^{\frac{1}{2}})^{\frac{1}{2}} \times 3(2)^{\frac{1}{2}} \times \left(\left(2^{\frac{1}{2}}\right)^{\frac{1}{2}} \times 2^{\frac{1}{2}}\right)^{\frac{1}{2}} \Rightarrow 2^{\frac{1}{4}} \times 3(2)^{\frac{1}{2}} \times (2)^{\frac{1}{4}} = 2^{\frac{1}{4}} \times 3 \times 2^{\frac{1}{4}} = 4 \times 3 = 12$

$$\frac{r^n (1 + r + r^2 + \dots + r^{n-1})}{r^n (\frac{1}{r} + \frac{1}{r^2} + \dots + 1)} = \left(\frac{r}{r}\right)^n \left(\frac{1 - \left(\frac{r}{r}\right)^n}{1 - \frac{1}{r}}\right) = \left(\frac{r}{r}\right)^n \left(\frac{r(r^n - 1)}{r^n - 1}\right) = r^n$$

$$\rightarrow \left(\frac{r}{r}\right)^n = r, r \rightarrow \boxed{n=r} \quad (a^r + b^r - rab)^r (a^r + b^r - rab)^r \rightarrow (a-b)^r (a+b)^r$$

$$\rightarrow (a-b)^r \rightarrow \left(\left(\sqrt{9}-r - \sqrt{9+r}\right)^r\right)^r \Rightarrow \sqrt{9}-r + \sqrt{9+r} - r\sqrt{r} = r\sqrt{9} - r\sqrt{r} \rightarrow \sqrt{14}-r$$

$$\left((r)(\sqrt{14}-r)\right)^r \rightarrow (r)(9+r-r\sqrt{r}) = (r)(1-r\sqrt{r}) \Rightarrow (r)(r)(r-\sqrt{r}) \rightarrow 19(r-\sqrt{r})$$

$$\rightarrow \boxed{t=19}$$

$$\sqrt[r]{r-r\sqrt{r}} \rightarrow \sqrt[r]{(r-\sqrt{r})^r} = \sqrt[r]{r-\sqrt{r}} = a$$

$$\left(\left(a + \frac{1}{a} + \sqrt{r}\right)\left(a + \frac{1}{a} - \sqrt{r}\right)\right)^r \rightarrow \left(a^r + \frac{1}{a^r} + r - r\right)^r = a^r + \frac{1}{a^r} + r = r^t$$

$$\rightarrow a^r = (r-\sqrt{r})^r + \frac{1}{(r-\sqrt{r})^r} + r \Rightarrow r - r\sqrt{r} + \frac{1}{r-\sqrt{r}} + r \rightarrow r - r\sqrt{r} + r + r\sqrt{r} + r = 19$$

$$19 = r^r = r^t \rightarrow \boxed{t=r}$$

$$A = \left(r^r \times r^{\frac{r}{r}}\right)^{\frac{1}{r}} \times r^{\frac{r}{r}} \rightarrow \left(r^{\frac{1}{r}}\right)^{\frac{1}{r}} = r^{\frac{1}{r^2}} \rightarrow r^{\frac{r}{r^2}} \times r^{\frac{r}{r^2}} = r^r = r = A$$

$$(r \times r)^{-\frac{1}{r}} = r^{-\frac{1}{r}} = \left(\frac{1}{r}\right)^{\frac{1}{r}} = \frac{1}{\sqrt[r]{r}} = \boxed{\frac{1}{r}}$$

$$\sqrt[r]{a} = r \times a^{\frac{1}{r}} \rightarrow a = \left(r^r \times a^{\frac{1}{r}}\right)^r \rightarrow a = r^r \times a^{\frac{1}{r}} \rightarrow \frac{1}{a^{\frac{1}{r}}} = r^r \rightarrow \frac{1}{a^r} = r^r$$

$$a^r = \frac{1}{r^r} = a = \frac{1}{r\sqrt{r}} \rightarrow \left(\frac{1}{a} - r\right) = r\sqrt{r} - r = r(\sqrt{r}-1) \rightarrow \frac{(r)(\sqrt{r}-1)}{1+\sqrt{r}} \times \frac{1-\sqrt{r}}{1-\sqrt{r}}$$

$$\frac{(r)(-1)(r+1-r\sqrt{r})}{-r} = \frac{(r)(r\sqrt{r}-r)}{-r} = \frac{(r)(r)(\sqrt{r}-1)}{-r} = -(r)(\sqrt{r}-1) = \boxed{-r\sqrt{r}+r}$$

$$\sqrt{m+a} = p \rightarrow p^r = m+a \rightarrow p^r - q^r = (p+q)(p^{r-1} - p^{r-2}q + \dots - q^{r-1}) = m+a - m+f$$

$$\sqrt{m-f} = q \rightarrow q^r = m-f$$

$$r(p+q) = r+a \rightarrow p+q = \frac{r+a}{r} \rightarrow \sqrt{m+a} + \sqrt{m-f} - r = p+q - r = \frac{r+a}{r} - r = \frac{a+f-f}{r} = \boxed{\frac{a}{r}}$$