بوپال ریاضی

۱) $(۱! + ۳!)(۲! + ۴!) = (۱+۶)(۲+۲۴) = ۱۸۲$

رقم یکان = $\boxed{۲}$

۲) الف) $\frac{۲۳+۸\sqrt{۷}}{۹} = \frac{(۴+\sqrt{۷})^{۲}}{۱۶-۷} = \frac{۴+\sqrt{۷}}{۴-\sqrt{۷}} = ۱+\sqrt{۷}$

$(۴+\sqrt{۷})^{-\frac{۱}{۴}}(۱+\sqrt{۷})^{\frac{۱}{۲}}$

$(۴+\sqrt{۷})^{-\frac{۱}{۴}} \times (۴+\sqrt{۷})^{\frac{۱}{۴}} = \boxed{۱}$

ب) $\frac{\sqrt{۲}+\sqrt{۵}}{\sqrt{۲}(\sqrt{۵}+\sqrt{۲})}\left(\frac{۳-\sqrt{۵}-۳-\sqrt{۵}}{\sqrt{۳-\sqrt{۵}}+\sqrt{۳+\sqrt{۵}}}\right) = \frac{۱}{\sqrt{۲}}\left(\frac{۲\sqrt{۵}}{\sqrt{۳-\sqrt{۵}}+\sqrt{۳+\sqrt{۵}}}\right)$

۳) الف) $\frac{\sqrt{۸}+\sqrt{۲۷}}{۵-\sqrt{۶}} = \frac{۲^{\frac{۳}{۲}}+۳^{\frac{۳}{۲}}}{۵-\sqrt{۶}}$ $\quad ۲(\sqrt[۴]{۹}-۱)^{-۱} = \frac{۲}{\sqrt{۳}-۱}$

$\sqrt{۸}+\sqrt{۲۷}\sqrt{۸-۲۷} \Rightarrow \sqrt{۸-۲۷}-\sqrt{۵+۶} \Rightarrow (۸-۲۷)-(۵+۶) = -۳۰$

$\Rightarrow \boxed{\sqrt{-۳۰}}$

ب) $\frac{۲\sqrt{۳}-۱}{۴+\sqrt{۳}} + \frac{۱}{۲-\sqrt{۳}} =$

۴) الف) $\left(\sqrt{۱+\sqrt{۳}}+\sqrt{\sqrt{۳}-۱}\right)^{۲} = (۱+\sqrt{۳})+\sqrt{۳}-۱+\sqrt{(۱+\sqrt{۳})(\sqrt{۳}-۱)}$

$= ۲\sqrt{(۱+\sqrt{۳})(\sqrt{۳}-۱)}+۲\sqrt{۳} \Rightarrow (۱+\sqrt{۳})(\sqrt{۳}-۱) = ۲$

$\Rightarrow ۲\sqrt{۲}+۲\sqrt{۳} \Rightarrow \left(\sqrt{۲}\sqrt{\sqrt{۲}+\sqrt{۳}}\right)\left(\frac{\sqrt{\sqrt{۲}+\sqrt{۳}}}{\sqrt{\sqrt{۳}-\sqrt{۲}}}\right) = ۱$

$۲-۲=۰ \Rightarrow$ جواب = ۲ کل کسر

ب) $\sqrt[۴]{۲\sqrt{۲^{۸}}} = (۲^{۸})^{\frac{۱}{۱۲}} = ۲^{\frac{۸}{۱۲}} = ۲^{\frac{۲}{۳}}$ $\quad \sqrt[۴]{۱۶۲} = (۲\times۳^{۴})^{\frac{۱}{۴}} = ۲^{\frac{۱}{۴}}\times۳$

$\sqrt{۴\times\sqrt[۶]{۲}} = (۴\times۲^{\frac{۱}{۶}})^{\frac{۱}{۲}} = ۲^{\frac{۳}{۱۲}}$

$۲^{\frac{۸}{۱۲}}\times۲^{\frac{۳}{۱۲}}\times۲^{\frac{۳}{۱۲}}\times۳ = ۲^{۲}\times۳ = \boxed{۱۲}$

۵)

$$\frac{3^x(1+3+3^2+3^3+3^4+3^5)}{2^{x-2}(1+2+2^2+2^3+2^4+2^5)} = \frac{3^x\times \cancel{364}}{2^{x-2}\times \cancel{63}\,9} = \frac{3^x}{2^{x-2}}\times\frac{52}{9} = 52$$

$$\Rightarrow 3^x = 9\times 2^{x-2} \Rightarrow 3^{x-2} = 2^{x-2} \Rightarrow \boxed{x=2}$$

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۶)

$$(a-b)^2(a+b)^2 = (a^2-b^2)^2 = t(2-\sqrt3) = \sqrt{\sqrt6+2} + \cancel{\sqrt{\sqrt6-2}} - \cancel{\sqrt{\sqrt6-2}}$$

$$t(2-\sqrt3) = \sqrt{\sqrt6+2} \qquad t = \frac{\sqrt{\sqrt6+2}}{2-\sqrt3}$$

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۷)

$$(K+\sqrt2)^2(K-\sqrt2)^2 = (K^2-2)^2 = \left(a^2+\frac{1}{a^2}+\cancel{2}-\cancel{2}\right)^2 \qquad a^2=\sqrt{(2-\sqrt3)^2} \qquad \boxed{a+\frac1a=K}$$

$$\Rightarrow 2-\sqrt3+\frac{1}{2-\sqrt3} = \frac{8-4\sqrt3}{2-\sqrt3} = \frac{4\cancel{(2-\sqrt3)}}{\cancel{2-\sqrt3}} \qquad 4^2 = 2^t \Rightarrow t=4$$

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۸)

$$4\times\sqrt[3]{16} = 2^2\times 2^{\frac43} = 2^{\frac{10}{3}} \Rightarrow \sqrt[5]{2^{\frac{10}{3}}} = 2^{\frac23}$$

$$\left(\frac12\right)^{-\frac43} = 2^{\frac43} \Rightarrow 2^{\frac23}\times 2^{\frac43} = 2^2 = 4 \Rightarrow A=4 \qquad (2A)^{-\frac13} = 8^{-\frac13} = \frac12$$

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۹)

$$\frac{a^{\frac17}}{a^{\frac{7}{15}}} = 27 \Rightarrow a^{-\frac{14}{7}} = 27 \Rightarrow a = \frac{1}{3\sqrt3}$$

$$\frac1a - 3 = 3\sqrt3 - 3 = 3(\sqrt3-1)$$

$$\frac{3(\sqrt3-1)}{\sqrt3+1} = \frac{3(3+1-2\sqrt3)}{4}$$

$$\frac{12-6\sqrt3}{4} = \boxed{\frac{6-3\sqrt3}{2}}$$

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۱۰)

$$\sqrt{x+a}+\sqrt{x-4}-2 = \cancel{2}+2\sqrt{x-4}\cancel{-2} = 2\sqrt{x-4}$$