پارسا سماعی دهم پسر ۸ تألیف ۲۲

۱

$x^2-5x+2=0$

$5=\alpha+\beta\ ,\ \alpha\beta=2$ $\qquad \beta(5-\beta)=2 \qquad \beta^2=5\beta-2$

$(\alpha+\beta)^2=25=\alpha^2+\beta^2+\underbrace{2\alpha\beta}_{4}\Rightarrow\alpha^2+\beta^2=21$

$$\frac{\overset{\alpha^2\beta^2}{4}\alpha+\beta^5}{\alpha\beta^2}=\frac{\alpha^3\beta^2+\beta^5}{(\beta+\alpha)\beta^2}=\frac{\beta^2(\alpha^3+\beta^3)}{(\beta+\alpha)\beta^2}$$

$$\frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{(\alpha+\beta)}=21-2=19$$

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$\frac{3+(-1.5)}{2}=0.75 \qquad -\frac{b}{2a}=\frac{3}{4} \qquad \alpha+\beta=-\frac{b}{a}=\frac{3}{2}$

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$x^2+2kx+5=0$

$\alpha-\beta=\frac{4}{3}k=\frac{\sqrt{\Delta}}{|a|}=\sqrt{\Delta}\Rightarrow\sqrt{4k^2-20}=\frac{4}{3}k\Rightarrow 4k^2-20=\frac{16}{9}k^2\Rightarrow k^2-5=\frac{4}{9}k^2$

$\frac{5}{9}k^2=5\Rightarrow k^2=9$

$\left[\frac{9}{2}\right]=[4.5]=4$

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$\frac{3+(-5)}{2}=-1=\frac{-b}{2a} \qquad \alpha+\beta=\frac{-b}{a}=-2\xrightarrow{\text{توان ۲}}\underbrace{\alpha^2+\beta^2}_{5}+2\alpha\beta=4\Rightarrow 2\alpha\beta=-1\Rightarrow\alpha\beta=-\frac{1}{2}=\frac{c}{a}$

$-\frac{\Delta}{4a}=1=\frac{-b^2+4ac}{4a}=\frac{-4a^2+4\times a\times-\frac{a}{2}}{4a}=\frac{-6a^2}{4a}=-\frac{3a}{2}=1\Rightarrow a=-\frac{2}{3}$ $\qquad b=2a \qquad c=-\frac{a}{2}$

نقطه‌ی عرض از مبدا $(0,\frac{1}{3})$ $\qquad \Rightarrow c=-\frac{-\frac{2}{3}}{\frac{2}{1}}=\frac{1}{3}$

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$-x^2+5x+m=0 \qquad x=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{5\pm\sqrt{25+4m}}{2}\Rightarrow\frac{5\pm\sqrt{25+4m}}{2}<\frac{9}{2}$

$5+\sqrt{25+4m}<9\Rightarrow 25+4m<16\Rightarrow m<-\frac{9}{4}=-2.25$

$5-\sqrt{25+4m}<9\Rightarrow-\sqrt{25+4m}<4\Rightarrow\sqrt{25+4m}>-4\Rightarrow\sqrt{25+4m}\geq 0\Rightarrow 25+4m\geq 0\Rightarrow 4m\geq-25\Rightarrow m\geq-6.25$

$\{-6,-5,-4,-3\}\Rightarrow$ ۴ مقدار صحیح

۶

$y=mx^2-12x+5m-1$

Min: $-\frac{b}{2a}=\frac{12}{2m}$; $-\frac{\Delta}{4a}=2\Rightarrow\Delta=-8m$

$12^2-4m(5m-1)=-8m$

$\Rightarrow 144-20m^2+4m=-8m$

$36-5m^2+m=-2m$

$\Rightarrow 5m^2-3m-36=0\Rightarrow m^2-3m-180=0$

$(m-15)(m+12)=0 \qquad m=3\ ✓\ ,\ m=-\frac{12}{5}=-2.4\ ✗$

$x=\frac{12}{2\times 3}=2$

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$x^2+2(a+1)x+2a-1=0 \qquad \alpha\beta=a^2=\frac{c}{a}\Rightarrow a^2=2a-1\Rightarrow a^2-2a+1=0\Rightarrow(a-1)^2=0\Rightarrow a=1$

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$x^4-7x^2-5=0\xrightarrow{x^2=t}t^2-7t-5=0 \qquad \Delta=49-4\times1\times-5=69$

$t=\frac{7\pm\sqrt{69}}{2}\Rightarrow\frac{7-\sqrt{69}}{2}$ ✗ چون منفی

$S=0 \qquad P=-\frac{7-\sqrt{69}}{2}$

$x=\pm\sqrt{\frac{7+\sqrt{69}}{2}}$

$2P^2-3SP+2S=\frac{49+69+14\sqrt{69}}{2}=59+7\sqrt{69}$

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$y=kx^2-4x-6 \qquad y=-4x-4\xrightarrow{x=\frac{2}{k}}-\frac{8}{k}-4$

$-\frac{b}{2a}=\frac{4}{2k}=\frac{2}{k}$

$y=\frac{4}{k}-\frac{8}{k}-6=-\frac{4}{k}-6$

$-\frac{8}{k}-4=-\frac{4}{k}-6\Rightarrow 2=\frac{4}{k}\Rightarrow k=2$

$y=-\frac{4}{2}-6=-8$

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$y=-mx^2+mx+1 \qquad y=-m-x \qquad -mx^2+mx+1=-m-x\Rightarrow mx^2-(m+1)x-(m+1)=0$

$\xrightarrow{\Delta<0}(m+1)^2+4m(m+1)<0\Rightarrow(m+1)(5m+1)<0$

	-1		$-\frac{1}{5}$	
+	0	−	0	+

$-\frac{1}{5}<m<-1$

هیچ مقدار صحیحی وجود ندارد ⇐ گ۳