نام و نام خانوادگی: سید محمدحسین اجاقی — پاسخنامه تشریحی تکلیف شماره ۳ — کلاس: دهم ریاضی A

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$y = ax^2 + bx + c \qquad \frac{-b}{2a} = -1 \qquad b = 2a$

$y = ax^2 + 2ax + c \quad \xRightarrow[x=-1]{y=9} \quad a - 2a + c = 9 \qquad -a + c = 9$

$y = ax^2 + 2ax + c \quad \xRightarrow[x=3]{y=1} \quad 9a + 6a + c = 1 \qquad 15a + c = 1$

$16a = -8 \Rightarrow a = -\frac{1}{2} \qquad c = \frac{17}{2}$

$$\boxed{-\frac{1}{2}x^2 - x + \frac{17}{2} = y}$$

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$2x^2 + mx + m + 6 = 0$

$\Delta > 0$	$\Delta = 0$	$\Delta < 0$
$\oplus\,\vert\,\oplus$	$\oplus$	$\times\,\vert\,\times$
$\oplus\,\vert\,\ominus$	$\ominus$	
$\ominus\,\vert\,\oplus$		
$\ominus\,\vert\,\ominus$		

۲ ریشه مثبت:

$\Delta > 0 \quad m^2 - 8m - 48 > 0 \quad (m+4)(m-12)$

	-4		12	
$+$	0	$-$	0	$+$

$(12, +\infty) \cup (-\infty, -4)$

$S > 0 \quad \frac{-b}{a} > 0 \Rightarrow \frac{-m}{2} > 0 \Rightarrow m < 0$

$P > 0 \quad \frac{c}{a} > 0 \Rightarrow \frac{m+6}{2} > 0 \Rightarrow m > -6$

$(-\infty, 0) \cap (-6, +\infty) \cap \left((12, +\infty) \cup (-\infty, -4)\right) = (-6, -4)$ بازه

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$3x^2 + (2m-1)x + 2 - m \qquad S = \frac{-b}{a} = \frac{-2m+1}{3} \qquad P = \frac{c}{a} = \frac{2-m}{3}$

$\frac{-2m+1}{3} = \frac{3}{2-m} \qquad 2m^2 - 5m + 2 = 9$

$2m^2 - 5m - 7 = 0 \xRightarrow{\text{روش}} m^2 - 5m - 14 = 0$

$(m-7)(m+2) \Rightarrow \begin{cases} m = -1 \Rightarrow 3x^2 - 3x + 3 = 0 \quad \Delta < 0 \quad \text{غ ق ق} \\ m = \frac{7}{2} \Rightarrow 3x^2 + 6x - \frac{3}{2} \quad \Delta > 0 \quad \text{ق ق} \end{cases}$

$m = \frac{7}{2}$

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$S = 1 \qquad P = -2 \qquad x^2 - x - 2 = 0$

یک مرحله‌ای کنیم

$\left(x - \left(x_2^3 + \frac{1}{x_1}\right)\right)\left(x - \left(x_1^3 + \frac{1}{x_2}\right)\right) = x^2 - \Big(\underbrace{x_1^3 + x_2^3}_{S^3 - 3SP} + \underbrace{\frac{1}{x_1} + \frac{1}{x_2}}_{\frac{S}{P}}\Big)x + \Big(\underbrace{x_1^3x_2^3}_{P^3} + \underbrace{x_2^2 + x_1^2}_{S^2 - 2P} + \underbrace{\frac{1}{x_1x_2}}_{\frac{1}{P}}\Big)$

$$\boxed{\Rightarrow x^2 - \frac{51}{4}x - \frac{221}{4} = 0}$$

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$\left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} + 1\right)\left(\sqrt[3]{x^2} - 1\right) = 2\sqrt[3]{x} \qquad \sqrt[3]{x^2} = t \Rightarrow x = t^{\frac{3}{2}}$

$\left(t + \frac{1}{t} + 1\right)(t - 1) = t^2 + 1 + t - t - \frac{1}{t} - 1 = 2\sqrt{t}$

$t^2 - \frac{1}{t} = 2\sqrt{t} \qquad \frac{t^3 - 1}{t} = 2t^{\frac{1}{2}}$

$t^3 - 1 = 2t^{\frac{3}{2}} \xRightarrow{x = t^{\frac{3}{2}}} x^2 - 2x - 1 = 0 \Rightarrow x = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$

$x_1 + x_2 = 1 + \sqrt{2} + 1 - \sqrt{2} = 2$

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$3(x-x_1)(x-3x_1) = 3x^2 - ax + 4 = 0$ $S=\frac{-b}{a}$ $P=\frac{c}{a}$

$\frac{+a}{3} = +4x_1$

$a = 12x_1$

$3x_1^2 = 4 \Rightarrow x_1 = \pm\sqrt{\frac{4}{3}} = \pm\frac{2\sqrt{3}}{3}$

$a = 12\times\frac{2\sqrt{3}}{3} = 8\sqrt{3}$ یا $12\times -\frac{2\sqrt{3}}{3} = -8\sqrt{3}$

$8\sqrt{3} - (-8\sqrt{3}) = 16\sqrt{3}$

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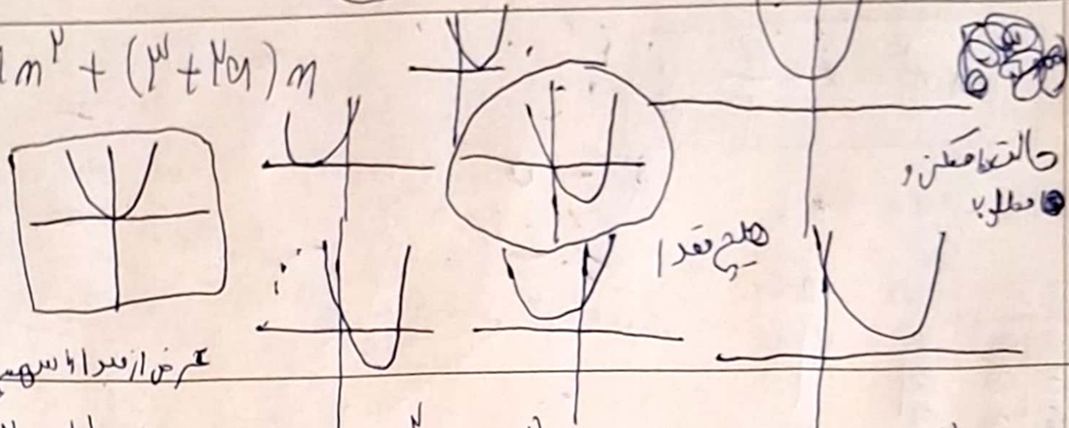
$y = ax^2 + (3+2a)x$

$a \neq 0$



عرض از مبدا سهمی صفر است

8

$-x^2 - 2x + b = y$ $\frac{+2}{-2} = -1$ $\boxed{x=-1}$

$y = x^2 + ax - 2$ $\frac{-a}{2} = -1$ $a = 2$

$y = x^2 + 2x - 2$

$x=-3$: $-x^2 - 2x + b = x^2 + 2x - 2 = 1 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow (x+3)(x-1) = 0$ $\begin{cases} x=1 \\ x=-3 \end{cases}$

$-9 + 6 + b = 1$ $b = 4$

$x=1$: $-1 - 2 + b = 1$ $b = 4$

$ab = 2\times 4 = 8$

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$2x^2 - ax + b = 0 \Rightarrow S = \frac{+a}{2}$

$2ax^2 + ax - 6 = 0 \Rightarrow S' = \underbrace{\frac{-a}{2a}}_{-0.5} = \frac{a}{2} - 1$

$\frac{a}{2} - \frac{1}{2} = 0$

$2x^2 + x - 6 = 0$

درس $\Rightarrow x^2 + x - 12 = 0$ $(x+4)(x-3) = 0$ $\Rightarrow \begin{cases} x = \frac{3}{2} \\ x = -2 \end{cases} \Rightarrow \begin{cases} x = 2 \\ x = -\frac{3}{2} \end{cases}$ $a = 1$

ریشه های معادله اول

$2x^2 - x + b = 0 = 2(x-2)(x+\frac{3}{2}) = 2x^2 - x - 6$ برابر نیم

$b = -6$ $a = 1$

$\left[\frac{ab}{4}\right] = \left[\frac{-6}{4}\right] = -2$

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$x^2 + 2x - 3m = 0$ $x^2 + 6x + m = 0 \Rightarrow \alpha^2 + 2\alpha - 3m = \alpha^2 + 6\alpha + m$

$\Rightarrow \alpha^2 + \underbrace{2\alpha + 3\alpha}_{5\alpha} = 0 \Rightarrow \alpha(\alpha+5) = 0 \Rightarrow \begin{cases} \alpha = 0 \times \text{غ ق ق} \\ \alpha = -5 \checkmark \end{cases}$

$\alpha^2 + 6\alpha - \alpha = 0$ $\alpha = 0 \times$ $\checkmark \alpha = -5$

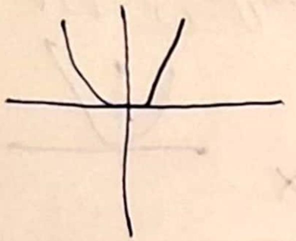
$-4\alpha = 4m$ $\alpha = -m$ $\Rightarrow m = 5$

$x^2 + 2x - 15 = 0$ $(x+5)(x-3) = 0$ $\begin{cases} x = -5 \\ x = 3 \end{cases}$

$x^2 + 6x + 5 = 0$ $(x+5)(x+1) = 0$ $\begin{cases} x = -1 \\ x = -5 \end{cases}$

$3 - (-1) = 4$

ادامه حل سوال ۷



$\alpha > 0$ $\quad$ $\alpha > 0 \Rightarrow \alpha \in \left(-\infty, -\frac{3}{2}\right]$

$P = 0$ همواره ✓ $\Rightarrow 3 + 2a \leq 0$

$S \geq 0$ $\quad$ $\frac{-b}{a} \geq 0 \Rightarrow b \leq 0$ چون $a>0$

$(0, +\infty) \cap \left(-\infty, -\frac{3}{2}\right] = \emptyset$ $\quad$ $\Delta \geq 0$ $\quad$ $(3+2a)^2 \geq 0$ ✓

همواره درست

به ازای هیچ مقدار