

$$\frac{-b}{2a} = -1 \Rightarrow -2a = -b \Rightarrow 2a = b \Rightarrow ax^2 + bx + c = ax^2 + 2ax + c$$

$$y = ax^2 + 2ax + c \begin{cases} 9 = a(-1)^2 + 2a(-1) + c \Rightarrow c - a = 9 \\ 1 = a(3)^2 + 2a(3) + c \Rightarrow 10a + c = 1 \end{cases} \Rightarrow 14a = -1 \Rightarrow a = -\frac{1}{14} \Rightarrow b = -\frac{1}{7}$$

$$c = \frac{14}{7} \Rightarrow c = 2$$

$$ax^2 + bx + c = -\frac{x^2}{14} - \frac{x}{7} + \frac{14}{7}$$

$$2x^2 + mx + m + 4 = 0 \Rightarrow \Delta > 0 \Rightarrow m^2 - 4(2)(m+4) > 0 \Rightarrow m^2 - 8m - 32 > 0 \Rightarrow (m-12)(m+4) > 0$$

$$S > 0 \Rightarrow \frac{-m}{2} > 0 \Rightarrow m < 0$$

$$P > 0 \Rightarrow \frac{m+4}{2} > 0 \Rightarrow m > -4$$

$$m = \mathbb{R} - [-4, 12]$$

$$(m < 0) \cap (m > -4) \cap (\mathbb{R} - [-4, 12]) = (-4, -4)$$

$$3x^2 + (2m-1)x + 2-m = 0$$

فرض می کنیم ریشه ها α و β هستند.

$$\alpha + \beta = \frac{1}{d\beta} \Rightarrow \frac{1-2m}{3} = \frac{1}{\beta} \Rightarrow \frac{1+2m}{3} = \frac{\beta}{\beta-m}$$

$$\Rightarrow 9 = 2-m-2m+2m^2 \Rightarrow 2m^2-4m-7=0 \Rightarrow m^2-2m-14=0 \Rightarrow m = \frac{2 \pm \sqrt{4+56}}{2} = 1 \pm \sqrt{15}$$

$$3x^2 - 3x + 3 = 0 \Rightarrow x^2 - x + 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

$$x = x^2 - 1 \Rightarrow x^2 - x - 1 \Rightarrow S_1 = 1 \text{ و } P_1 = -1$$

$$S_p = \frac{x_p^3 + 1}{x_1} + \frac{x_1^3 + 1}{x_p} = \frac{x_1^3 + x_p^3}{x_1 x_p} + \frac{x_1 + x_p}{x_1 x_p} = \frac{(1)^3 - 1^3(1)(-1)}{1^3} + \frac{1}{-1} = \frac{0}{1} - 1 = -1$$

$$P_p = \left(\frac{x_p^3 + 1}{x_1} \right) \left(\frac{x_1^3 + 1}{x_p} \right) = \frac{(x_1 x_p)^3 + x_1^3 + x_p^3 + 1}{x_1 x_p} = \frac{-4 + ((1)^3 - 1^3(-1))}{-1} = \frac{-4 + 0}{-1} = 4$$

$$x^2 - \frac{0}{1}x - \frac{4}{1} = 0 \Rightarrow x^2 - 4 = 0 \Rightarrow x = \pm 2$$

$$\left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} + 1 \right) \left(\sqrt[3]{x^2} - 1 \right) = \sqrt[3]{x^2} \xrightarrow{\sqrt[3]{x^2} = t} \left(t^2 + \frac{1}{t^2} + 1 \right) (t^2 - 1) = t^2$$

$$= t^4 - t^2 + 1 - \frac{1}{t^2} + t^2 - 1 = t^4 - \frac{1}{t^2} = 0 \xrightarrow{x=t^2} t^6 - 1 = 0 \Rightarrow t^6 = 1 \Rightarrow t = \sqrt[6]{1} = 1$$

$$t = \sqrt[3]{x} \Rightarrow x^2 - 2x - 1 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$$

$$x_1 + x_2 = S = \frac{b}{a} = \frac{2}{1} = 2$$

۶- فرض می‌کنیم ریشه‌ها β و β^r هستند:

$$P = \beta \times \beta^r = \beta^r = \frac{r}{\beta} \Rightarrow \beta^r = \frac{r}{\beta} \Rightarrow \beta = \pm \frac{r}{\beta}$$

$$S = \beta + \beta^r \begin{cases} \frac{1}{\beta} \Rightarrow \frac{a_1}{\beta} = \frac{1}{\beta} \Rightarrow a_1 = 1 \\ -\frac{1}{\beta} \Rightarrow \frac{a_2}{\beta} = -\frac{1}{\beta} \Rightarrow a_2 = -1 \end{cases} \quad a_1 - a_2 = 14$$

۷- چون سهمی عرض از مبدأ ندارد پس یکی از ریشه‌ها در Δ مثبت است و نیاز به بررسی ندارد.

$$a > 0 \quad S = -\frac{r+a}{a} \geq 0 \quad -\frac{r}{a} + \frac{0}{a} \rightarrow a = [-\frac{r}{a}, 0) \rightarrow (a > 0) \cap [-\frac{r}{a}, 0) = \emptyset$$

$$\begin{cases} y = x^2 + ax - 2 \\ y = -x^2 - 2x + b \end{cases} \quad \frac{y}{-2} = -\frac{a}{2} \Rightarrow a = 2$$

$$1 = x^2 + 2x - 2 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow (x-1)(x+3) = 0$$

چون دو نقطه از هر دو سهمی روی خط $y=1$ هستند در نتیجه این دو نقطه در دو سهمی مشترک بوده و در نتیجه سهمی $y = -x^2 - 2x + b$ باید به ازای $x=1$ و $x=-3$ ، $y=1$ تحویل بدهد:

$$1 = -9 + 4 + b \Rightarrow b = 4 \Rightarrow ab = 2 \times 4 = 8$$

$$2x^2 - ax + b = 0 \quad 2ax^2 + ax - 4 = 0$$

$$\alpha, \beta \quad \alpha - \frac{1}{2}, \beta - \frac{1}{2}$$

$$a + \beta = \frac{a}{2}$$

$$a\beta = \frac{b}{2}$$

$$\frac{a}{2} \leftarrow \frac{a}{2} \quad a + \beta - 1 = -\frac{1}{2}$$

$$(\alpha - \frac{1}{2})(\beta - \frac{1}{2}) = \alpha\beta + \frac{1}{4} - \frac{\alpha + \beta}{2} = -\frac{3}{4}$$

$$\frac{a}{2} - 1 = -\frac{1}{2} \Rightarrow a = 1$$

$$\left[\frac{ab}{2}\right] = \left[\frac{-4}{2}\right] = -2 \quad \leftarrow \frac{b}{2} + \frac{1}{4} - \frac{1}{4} = -3 \Rightarrow b = -4$$

$$x^2 + 2x - 3m = x^2 + 4x + m \Rightarrow 2x + 4m = 0 \Rightarrow x = -2m \Rightarrow m \neq 0$$

پس یعنی هر دو معادله باید به ازای $x = -2m$ صفر بدهند.

$$x = -2m \Rightarrow m^2 - 2m - 3m = 0 \Rightarrow m(m-5) = 0 \Rightarrow m \in \{0, 5\}$$

$$x^2 + 2x - 15 = 0$$

$$(x+5)(x-3) = 0 \Rightarrow x \in \{-5, 3\}$$

$$x^2 + 4x + 5 = 0$$

$$(x+1)(x+5) = 0 \Rightarrow x \in \{-1, -5\}$$

$$3 - (-1) = 4$$