

$$y = a(x - h)^2 + k \quad (h, k) = (-1, 9) \quad 1$$

$$y = a(x + 1)^2 + 9 \quad 3 = a(1 + 1)^2 + 9 \rightarrow 3 = 4a + 9$$

$$a = -\frac{1}{2} \quad y = -\frac{1}{2}(x + 1)^2 + 9 \quad y = -\frac{1}{2}x^2 - x + \frac{9}{2}$$

$$y = -\frac{1}{2}x^2 - x + \frac{9}{2}$$

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$$\Delta = m^2 - 4(2)(m + 5) \geq 0 \quad (m - 12)(m + 4) \geq 0$$

10

$$x_1 \times x_2 = \frac{m + 5}{2} > 0 \quad m > -5 \quad x_1 + x_2 = -\frac{m}{2} > 0$$

12

$$m \geq 12 \text{ و } m \leq -4 \quad m > -5 \quad m < 0 \quad m \in (-5, -4]$$

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$$m^2 - 8m - 20 = 0 \quad \checkmark m = \frac{4}{2} \quad \times m = -1 \quad \Delta \geq 0$$

15

$$(m - 1)^2 - 4(2)(2 + m) \geq 0 \quad m^2 + 12m - 23 > 0$$

16

$$m = \frac{-12 \pm \sqrt{144 + 4 \times 23}}{2} = \frac{-12 \pm 14\sqrt{3}}{2} = \frac{-12 \pm 14\sqrt{3}}{2}$$

18

$$m \leq \frac{-12 - 14\sqrt{3}}{2} \quad m \geq \frac{-12 + 14\sqrt{3}}{2}$$

20

$$f(1) + 1(-1) - 23 = 4 - 1 - 23 = -20 < 0 \quad m = -1 \quad \times$$

$$f\left(\frac{4}{2}\right) + 1\left(\frac{4}{2}\right) - 23 = 4 + 2 - 23 = -17 < 0 \quad m = \frac{4}{2} \quad \checkmark$$

25

$$x_1^2 = 2x_1 + 4x_2^2 = 2x_2 + 4 \quad \left(x_1^2 + \frac{1}{x_2}\right) + \left(x_2^2 + \frac{1}{x_1}\right) =$$

$$(2x_1 + 4) + (2x_2 + 4) + \frac{1}{x_2} + \frac{1}{x_1} = 2(1) + 4 + \frac{1}{-4} =$$

29

$$1 - \frac{1}{4} = \frac{3}{4} \quad \left(x_1^2 + \frac{1}{x_2}\right) \times \left(x_2^2 + \frac{1}{x_1}\right) = x_1^2 x_2^2 + x_1^2 \times \frac{1}{x_1}$$

30

$$\frac{1}{x_1} + x_1^2 \times \frac{1}{x_2} + \frac{1}{x_1 x_2} = (x_1 x_2) + x_1^2 + x_2^2 + \frac{1}{x_1 x_2}$$

31

$$(4) + (x_1 + x_2)^2 - 2x_1 x_2 + \frac{1}{-4} = -4 + 1 + 4 - \frac{1}{4} = -\frac{1}{4}$$

$$x_1^2 - \frac{1}{4} - \frac{1}{x_2} = 0 \quad (4x - 1 - 221) = 0 \quad \text{نادر}$$

$$t = \sqrt{x^2} \quad (t + \frac{1}{t} + 1)(t - 1) = 2\sqrt{x^2} \quad -5$$

$$\sqrt{x} = \sqrt{\frac{x^2}{x}} = \frac{t}{\sqrt{x}} \quad u = \sqrt{x}$$

$$(u^2 + \frac{1}{u^2} + 1)(u^2 - 1) = 2u \quad \xrightarrow{\text{ضرب}} (u^4 + 1 + u^2)(u^2 - 1) = 2u^3$$

$$u^6 + u^4 - u^2 - 1 = 2u^3 \quad u^6 - 2u^3 + u^4 - u^2 - 1 = 0$$

$$(u^2 - 1)(u^4 + u^2 + 1) = 0 \quad u^2 - 1 = 0 \quad u = 1$$

ملاحظه کنید: $u^4 + u^2 + 1 = 0$ (درستی محقق)

$$x_1 = 1 \quad x_2 = 0 \quad x + 1 = 9$$

$$r + 3r = \frac{9}{4} \quad a = 12r$$

$$r \times r = \frac{9}{4} \quad r^2 = \frac{9}{4} \quad r = \frac{3}{2} \quad a_1 = 12 \times \frac{3}{2} = 18$$

$$r = -\frac{3}{2} \quad a_2 = 12 \times (-\frac{3}{2}) = -18$$

$$|a_1 - a_2| = |18 - (-18)| = |36| = 36$$

$$y = ax^2 + (3 + 2a)x = x[ax + (3 + 2a)] \quad x_1 = 0$$

$$x_2 = -\frac{3+2a}{a} \quad y < 0 \text{ و } x < 0 \Rightarrow a < 0$$

$$-\frac{3+2a}{a} > 0 \quad \frac{3+2a}{a} < 0 \quad 3+2a > 0 \quad a < -\frac{3}{2}$$

$$-\frac{3}{2} < a < 0 \quad a \in (-\frac{3}{2}, 0)$$

$$x = -\frac{a}{2} \quad x = -1 \quad -\frac{a}{2} = -1 \quad y = k$$

$$k = x_1^2 + 2x_1 - 2 \quad k = -x_2^2 - 2x_2 + b$$

$$y = 1 \quad k = 1 \quad 1 = x^2 + 2x - 2 \quad x^2 + 2x - 3 = 0 \quad x_1 = -3, x_2 = 1$$

$$1 = -x^2 - 2x + b \quad x^2 + 2x + 1 = b \quad x_2 = -1 \pm \sqrt{b-2}$$

$$x_1 = -3, x_2 = 1 \quad x_2 = 1 \quad (1+1)^2 = b \quad b = 4 \quad x_2 = -3$$

$$(-3+1)^2 = b \quad b = 4 \quad a = 2 \quad 2 \times 2 = 4$$

$$x_1 = r_1 + \frac{1}{r_1}, x_2 = r_2 + \frac{1}{r_2} \quad r_1 + r_2 = -\frac{a}{2} = -1$$

$$r_1 r_2 = -\frac{b}{a} = -2 \quad x_1 + x_2 = (r_1 + \frac{1}{r_1}) + (r_2 + \frac{1}{r_2}) =$$

$$(r_1 + r_2) + \frac{1}{r_1 r_2} = -1 + \frac{1}{-2} = -\frac{3}{2} \quad \frac{a}{2} = \frac{1}{r_1 r_2} \Rightarrow \frac{2}{-2} = \frac{1}{-2}$$

$$x_1 x_2 = (r_1 + \frac{1}{r_1})(r_2 + \frac{1}{r_2}) = r_1 r_2 + \frac{1}{r_1 r_2} + \frac{1}{r_1} + \frac{1}{r_2}$$

$$-\frac{3}{2} - \frac{1}{2} + \frac{1}{-2} = -\frac{3}{2} \quad x_1 x_2 = -\frac{3}{2} \quad b = -\frac{3}{2}$$

$$\left[\frac{a}{2}\right] \left[\frac{b}{2}\right] = \left[\frac{2}{2}\right] \left[\frac{-3}{2}\right] = [-1, 5]$$

10- ریشه مشترک غیر صفری و یکم.

$$r^2 + 9r + m = 0 \quad (r^2 + 9r + m) - (r^2 + 2r - 3m) = 0$$

$$7r + 4m = 0 \quad r = -\frac{4m}{7}$$

$$(-\frac{4m}{7})^2 + 9(-\frac{4m}{7}) + m = 0 \quad m^2 - 5m = 0 \quad m(m-5) = 0 \quad m = 0$$

$$m \neq 0 \quad r = -5 \quad (x+5)(x-3) = 0$$

$$x = -5 \quad x = 3$$

$$|3 - (-5)| = |8| = 8$$