Subject: تکلیف شماره ۲۳ دهم تجربی A        Date: پارسا رجائی

**۱ـ**

$\frac{-b}{a} = -1 \rightarrow -2a = -b \Rightarrow 2a = b \Rightarrow ax^2 + bx + c = ax^2 + 2ax + c$

$y = ax^2 + 2ax + c \rightarrow 9 = a(-1)^2 + 2a(-1) + c \Rightarrow c - a = 9$

$\searrow 1 = a(3)^2 + 2a(3) + c \Rightarrow 15a + c = 1$

$\Rightarrow 16a = -8 \qquad a = -\frac{1}{2} \rightarrow b = -1 \qquad c = \frac{17}{2}$

$ax^2 + bx + c = \frac{-x^2}{2} - x + \frac{17}{2}$

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**۲ـ**

$2x^2 + mx + m + 6 = 0 \rightarrow \Delta > 0 \rightarrow m^2 - 4(2)(m+6) \Rightarrow m^2 - 8m - 48 > 0 \Rightarrow (m-12)(m+4) > 0$

| $x$ | | $-4$ | | $12$ | |
|---|---|---|---|---|---|
| $y$ | $+$ | | $-$ | | $+$ |

$m = R - [-4, 12] \qquad S > 0 \rightarrow \frac{-m}{2} > 0 \Rightarrow m \leq 0$

$P > 0 \rightarrow \frac{m+6}{2} > 0$

| $x$ | | $-6$ | |
|---|---|---|---|
| $y$ | $-$ | | $+$ |

$m > -6$

$(m < 0) \cap (m > -6) \cap (R - [-4, 12]) = (-6, -4)$

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**۳ـ**

$3x^2 + (2m-1)x + 2 - m = 0 \qquad \alpha + \beta = \frac{1}{\alpha\beta} \qquad \frac{1-2m}{3} = \frac{1}{\frac{2-m}{3}} \Rightarrow \frac{1-2m}{3} = \frac{3}{2-m}$

$\Rightarrow 9 = 2 - m - 4m + 2m^2 \Rightarrow 2m^2 - 5m - 7 = 0 \rightarrow m^2 - 5m - 14 = 0 \rightarrow (m-7)(m+2) = 0$

$\checkmark \quad \frac{7}{2}$

$3x^2 - 3x + 3 \qquad \Delta < 0 \quad \times \quad -1$

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**۴ـ**

$x = x^2 - 4 \Rightarrow x^2 - 4 - x = 0 \rightarrow S_1 = -1 \qquad P_1 = -4$

$S_2 = x_2^3 + \frac{1}{x_1} + x_1^3 + \frac{1}{x_2} = \underbrace{x_1^3 + x_2^3}_{S^3 - 3PS} + \frac{\overbrace{x_1 + x_2}^{S}}{\underbrace{x_1 x_2}_{P}} = \frac{51}{4}$

$P_2 = \left(x_2^3 + \frac{1}{x_1}\right)\left(x_1^3 + \frac{1}{x_2}\right) = (x_1 x_2)^3 + \underbrace{x_2^2 + x_1^2}_{S^2 - 2P} + \frac{1}{x_1 x_2} = \frac{-219}{4}$

$\rightarrow x^2 - \frac{51}{4}x - \frac{219}{4} = y$

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**۵ـ**

$\left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} + 1\right)\left(\sqrt[3]{x^2} - 1\right) = 2\sqrt[3]{x} \xrightarrow{\sqrt[3]{x} = t} \left(t^2 + \frac{1}{t^2} + 1\right)(t^2 - 1) = 2t$

$= t^4 - t^2 + 1 - \frac{1}{t^2} + t^2 - 1 = 2t \Rightarrow t^4 - \frac{1}{t^2} - 2t = 0 \xrightarrow{\times t^2} t^6 - 2t^3 - 1 + 2 = 0 \xrightarrow{t = \sqrt[3]{x}} x^2 - 2x - 1 = 0$

$x_1 + x_2 = S = \frac{-b}{a} = \frac{-(-2)}{1} = 2$

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**۶-**

$$3x^2-ax+4=0 \qquad \left.\begin{matrix} x_1=3\beta \\ x_2=\beta \end{matrix}\right\} \text{فرض می‌کنیم}$$

$$P=3\beta\times\beta=3\beta^2=\frac{4}{3}\Rightarrow \beta^2=\frac{4}{9}\Rightarrow \beta=\pm\frac{2}{3}$$

$$S=4\beta \longrightarrow \frac{8}{3}\Rightarrow \frac{a_1}{3}=\frac{8}{3}\Rightarrow a_1=8$$

$$\searrow -\frac{8}{3}\Rightarrow \frac{a_2}{3}=-\frac{8}{3}\Rightarrow a_2=-8 \qquad a_1-a_2=16$$

**۷-**

$$y=ax^2+(3+2a)x \quad a>0 \to (0,+\infty) \qquad S=-\frac{3+2a}{a}\geq 0$$

| $x$ | | $-\frac{3}{2}$ | | $0$ | |
|---|---|---|---|---|---|
| $y$ | $-$ | ¢ | $+$ | ¢ | $-$ |

$$\to a\in\left[-\frac{3}{2},0\right) \Longrightarrow (0,+\infty)\cap\left[-\frac{3}{2},0\right)=\varnothing$$

سهمی مورد نظر دارای عرض از مبدأ نمی باشد بنابراین یک یا هر ۲ ریشه‌ها صفر خواهند بود و $\Delta>0$ می باشد

**۸-**

$$\left.\begin{matrix} y=x^2+ax-2 \\ y=-x^2-2x+b \end{matrix}\right\} \longrightarrow \frac{2}{-2}=\frac{-a}{2}\Rightarrow a=2$$

$$1=x^2+2x-2\Rightarrow x^2+2x-3=0\to (x-1)(x+3)=0$$

$$y=-x^2-2x+b \xrightarrow{x=1 \; y=1} 1=-1-2+b\Rightarrow b=4 \qquad ab=2\times4=8$$

$$\xrightarrow[x=-3]{y=1} 1=-9+6+b\Rightarrow b=4$$

**۹-**

$$2x^2-ax+b=0 \qquad 2ax^2+ax-6=0$$

$$\alpha,\beta \qquad \alpha-\frac{1}{2},\ \beta-\frac{1}{2} \longrightarrow \frac{a}{2}-1=-\frac{1}{2}\Rightarrow \frac{a}{2}=\frac{1}{2}\Rightarrow a=1$$

$$\alpha+\beta=\frac{a}{2} \qquad \alpha+\beta-1=-\frac{1}{2}$$

$$\alpha\beta=\frac{b}{2} \qquad \left(\alpha-\frac{1}{2}\right)\left(\beta-\frac{1}{2}\right)=\underbrace{\alpha\beta}_{\frac{b}{2}}+\frac{1}{4}-\underbrace{\frac{\alpha+\beta}{2}}_{-\frac{1}{4}}=\underbrace{\frac{-3}{a}}_{-3}\to\frac{b}{2}+\frac{1}{4}-\frac{1}{4}=-3\Rightarrow b=-6$$

$$\left[\frac{ab}{4}\right]=\left[\frac{-6}{4}\right]=-2$$

**۱۰-**

$$x^2+2x-3m=x^2+6x+m\Rightarrow 4x+4m=0\longrightarrow x=-m$$

$$x^2+2x-3m=0\xrightarrow{x=-m} m^2-2m-3m=0\to m(m-5)\to m \to 0 \times,\ \ 5 \checkmark$$

$$x^2+6x+5=0\longrightarrow (x+1)(x+5)\to x=-1,\ x=-5 \qquad 3-(-1)=4$$

$$x^2+2x-15=0\longrightarrow (x+5)(x-3)\to x=3,\ x=-5$$