آرین عباسی

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$y = a(x-h)^2 + k \rightarrow a(x+1)^2 + 9$

جایگذاری

$\rightarrow 1 = a(3+1)^2 + 9 \rightarrow 1 = 16a + 9 \rightarrow -8 = 16a \rightarrow a = -\frac{1}{2}$

$-1 = 2a$

$y = -\frac{1}{2}(x+1)^2 + 9$

$y = -\frac{1}{2}(x^2 + 1 + 2x) + 9 \rightarrow \boxed{y = -\frac{1}{2}x^2 - x + \frac{17}{2}}$

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$2x^2 + mx + m + 6 = 0$ دو ریشه $\boxed{\Delta > 0}$

$\rightarrow \Delta = m^2 - 4(2)(m+6) \rightarrow m^2 - 8m - 48 > 0$

$\leftarrow (m-12)(m+4) > 0$

$\boxed{m > 12 \text{ یا } m < -4} \leftarrow$ $+ \quad -4 \quad - \quad 12 \quad +$

شرط ۲: $\frac{c}{a} = \frac{m+6}{2} > 0 \rightarrow m + 6 > 0 \rightarrow \boxed{m > -6}$

$-6 \quad -4 \quad 0 \quad 12$

شرط ۳: $x_1 + x_2 = \frac{-b}{a} = \frac{-m}{2} > 0 \rightarrow \boxed{m < 0}$

$\boxed{-6 < m < -4}$

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$3x^2 + (2m-1)x + (2-m) = 0$

$\frac{c}{a} = \frac{2-m}{3}$

$\frac{-b}{a} = -\frac{2m-1}{3}$

مجموع ریشه‌ها = معکوس حاصل ضرب ریشه‌ها

شرط سوال: $x_1 + x_2 = \frac{1}{x_1 x_2} \rightarrow -\left(\frac{2m-1}{3}\right) = \frac{1}{\frac{2-m}{3}} \Rightarrow -(2m-1)(2-m) = 9$

$-(-2m^2 + 5m - 2) = 9 \rightarrow$

$2m^2 - 5m + 2 = 9 \rightarrow 2m^2 - 5m - 7 = 0$

$\sqrt{\Delta} = (-5)^2 - 4 \times 2 \times -7 = 9$

$m_1 = \frac{14}{4} = \frac{7}{2}$

$m_2 = -1$

$\leftarrow m = \frac{5 \pm 9}{4}$

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$\frac{-b}{a} = \frac{1}{1} = 1$

$\frac{c}{a} = -4$

$(x_1^3 + x_2^3) + \left(\frac{1}{x_1} + \frac{1}{x_2}\right) = ?$

$(x_1 + x_2)^3 - 3x_1 x_2 (x_1 + x_2) \Rightarrow 1 + 12 = 13$

$\frac{x_2 + x_1}{x_1 x_2} \Rightarrow \frac{1}{-4}$

$13 - \frac{1}{4} = \frac{51}{4}$

مجموع ریشه‌ها

ضرب ریشه‌ها: $\left(x_1^3 + \frac{1}{x_2}\right)\left(x_2^3 + \frac{1}{x_1}\right) \Rightarrow x_1^3 x_2^3 + x_1^2 + x_2^2 + \frac{1}{x_1 x_2} = -64 + 9 - \frac{1}{4} = \frac{-221}{4}$

$(x_1 + x_2)^2 - 2x_1 x_2 = 1 + 8 = 9$

$\boxed{y = x^2 - \frac{51}{4}x - \frac{221}{4}}$

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$t=\sqrt[3]{x} \xrightarrow[\text{عبارت}]{\text{ساده شده}} \left(t^2+\frac{1}{t^4}+1\right)\left(t^2-1\right)=2t$

$\xrightarrow{t} (t^2-1)t^2+(t^2-1)\frac{1}{t^4}+(t^2-1) \Rightarrow (t^4-t^2)+\left(1-\frac{1}{t^2}\right)+(t^2-1) \Rightarrow$

$t^4-\frac{1}{t^2}=2t \xrightarrow{\text{ضرب در } t^2} t^6-2t^3-1=0 \xrightarrow{t^3=u} u^2-2u-1=0$

$u=\frac{2\pm\sqrt{4+4}}{2}=\frac{2\pm\sqrt{8}}{2}=1\pm\sqrt{2} \rightarrow t^3=1+\sqrt{2} \rightarrow t=\sqrt[3]{1+\sqrt{2}}$

$t^3=1-\sqrt{2} \rightarrow t=\sqrt[3]{1-\sqrt{2}}$

$x=t^3 \rightarrow x \begin{cases}1+\sqrt{2}\\1-\sqrt{2}\end{cases}$

$x_1+x_2=1+\sqrt{2}+1-\sqrt{2}=2$

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$\frac{-b}{a}=\frac{a}{3} \quad , \quad \frac{c}{a}=\frac{4}{3}$

شرط سوال: $x_1=3x_2 \rightarrow x_1+x_2=3x_2+x_2=4x_2=\frac{a}{3} \Rightarrow x_2=\frac{a}{12} \quad , \quad x_1=\frac{a}{4}$

ضرب: $\frac{a}{4}\times\frac{a}{12}=\frac{a^2}{48} \rightarrow \frac{a^2}{48}=\frac{4}{3} \rightarrow a^2=64 \rightarrow a\begin{cases}8\\-8\end{cases}$ $\quad |8-(-8)|=16$ اختلاف

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$\rightarrow a>0$

$x, y<0$

ریشه‌ها $\rightarrow ax^2+(3+2a)x \rightarrow x(ax+3+2a)$

(زیر محور) $y<0 , x<0$

$y>0_{x<0} \leftarrow$ اگر $\rightarrow x(ax+3+2a)\geq 0 \xrightarrow{\text{برای همه}} x<0 \quad \begin{cases}x=0\\x=-\frac{3+2a}{a}\end{cases}$ ریشه‌ها

$y>0 \quad \checkmark\ x_2<0 \quad : a>0 \qquad -\frac{3+2a}{a}<0 \quad x_2<0$ $\leftarrow$ بدون علامت :

$x\rightarrow -\infty$

$\boxed{-\frac{3}{2}<a<0} \quad 3+2a>0 \leftarrow a<0$

$a>-\frac{3}{2}$

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$1=x^2+ax-3 \rightarrow x^2+ax-3=0$

$t=-x^2-2x+b \rightarrow -x^2-2x+b-1=0 \rightarrow x^2+2x-b+1=0$

$x^2+ax-3=x^2+2x-b+1 \rightarrow ax-3=2x-b+1 \rightarrow (a-2)x=2-b$

$a-2=0 \rightarrow a=2$

$0=2-b \Rightarrow b=2$

$ab=2\times2=4$

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ریشه‌های معادلهٔ اول: x_1 , x_2

ریشه‌های معادلهٔ دوم: $x_1+\frac{1}{2} , x_2+\frac{1}{2}$

$$\begin{cases} 2ax^2+ax-6=0 \\ \text{جمع ریشه‌ها}: \frac{-b}{a} \to x_1+x_2+\left(\cancel{\frac{1}{2}+\frac{1}{2}}\right) = \frac{-a}{2a} = -\frac{1}{2} \to \underbrace{x_1+x_2}_{-\frac{3}{2}}+1=\frac{-1}{2} \end{cases}$$

معادلهٔ دوم

$0=2x^2-ax+b$

$$\frac{-b}{a} = \frac{a}{2} = \frac{-3}{2} \Rightarrow a=-3 \quad , \quad x_1x_2=\frac{b}{2}$$

ضرب: $\left(x_1+\frac{1}{2}\right)\left(x_2+\frac{1}{2}\right) = x_1x_2+\frac{(x_1+x_2)}{2}+\frac{1}{4} = ? \to x_1x_2-\frac{3}{4}+\frac{1}{4} = x_1x_2$

$$\left(x_1+\frac{1}{2}\right)\left(x_2+\frac{1}{2}\right) = \frac{-6}{2a} = \frac{-6}{2\times -3} = -1 \to x_1x_2-\frac{1}{2}=1 \to x_1x_2=\frac{3}{2}$$

$\frac{-1}{2}$

$$b \to x_1x_2=\frac{b}{2} \to \frac{3}{2} \to b=3 \to \left[\frac{ab}{4}\right] \to \frac{-3\times 3}{4} \to \left[\frac{-9}{4}\right] = -3$$

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ریشهٔ مشترک $=K$

معادلهٔ اول: $K^2+2K-3m=0 \to 3m=K^2+2K \to m=\frac{K^2+2K}{3}$

معادلهٔ دوم: $\to K^2+6K+m=0 \to m=-(K^2+6K)$

$$\frac{K^2+2K}{3} = -(K^2+6K) \to -3K^2-18K=K^2+2K \to 4K^2+20K=0$$

$$4K(K+5)=0$$

$\boxed{K=-5}$ ، $\cancel{K=0}$ غ ق (فرض مسئله) غیر صفر

$$m=\frac{K^2+2K}{3} \to \frac{(-5)^2+2(-5)}{3} = \frac{25-10}{3} = 5$$

$$x^2+2x-3m = x^2+2x-15=0$$

$x_1=K=-5$ ، $\frac{-b}{a}=-2 \to -5+x_2=-2$ ، $\underline{x_2=3}$

$$x^2+6x+m = x^2+6x+5=0 \to x_1=-5$$

$$\frac{-b}{a}=-6 \to -5+x_2=-6 \to x_2=-1$$

اختلاف: $3-(-1)=4$