مهدی پیمان شاکری — تکلیف شماره ۲۴ — کلاس دهم ریاضی A

۲۰

۱) الف)

$y = 2x^2 - 4x + 1 \quad a > 0 \rightarrow$ min دارد

$\min \left| \begin{matrix} \frac{-b}{2a} = 1 \\ = -1 \end{matrix} \right.$ ✓

$\min(1, -1)$

(۲)

ب) $y = -2x^2 + 3x - 5 \quad a < 0 \rightarrow$ max دارد

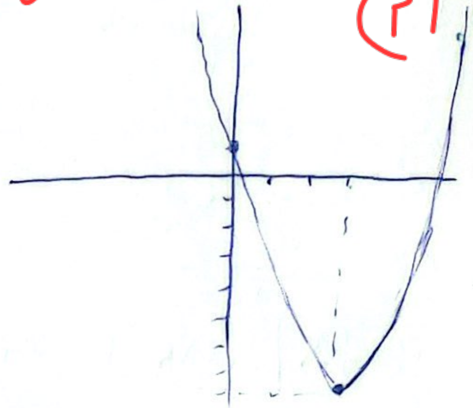
$\max \left| \begin{matrix} \frac{-b}{2a} = \frac{-3}{-4} = \frac{3}{4} \\ \frac{-\Delta}{4a} = -\frac{31}{8} \end{matrix} \right.$ ✓

۲) (۲)

$y = x^2 - 6x + 1$ (عرض از مبدا)

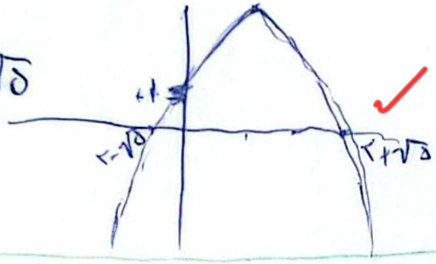
$a > 0 \rightarrow$ min دارد

$x_{راس} = \frac{6}{2} = 3 \quad y_{راس} = -8$ ✓



ب) $y = -x^2 + 4x + 1 \quad a < 0 \rightarrow$ max دارد

$\max \left| \begin{matrix} 2 \\ 5 \end{matrix} \right.$ ✓ $\quad x_{1,2} = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}$



۳) (۲)

$4x^3 + kx^2 - 9x - 2 = 0 \quad \alpha + \beta = 1 \quad \alpha\beta = -2$

$x^2 - Sx + P = y \rightarrow$ یکی از عامل‌ها $\rightarrow (x^2 - x - 2)(4x + a) = 4x^3 + kx^2 - 9x - 2$

$4x^3 + (a-4)x^2 + (-a-8)x - 2a \longrightarrow a - 4 = k \quad -a - 8 = -9 \quad -2a = -2$

$a = 1$

$a - 4 = k \longrightarrow k = 1 - 4 = -3$ ✓

$\alpha+\beta=1 \rightarrow \alpha=1-\beta \quad \alpha\beta=-2 \rightarrow (1-\beta)\beta=-2 \quad -\beta^2+\beta+2=0$

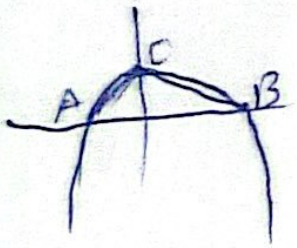
$\rightarrow \beta \begin{cases} \nearrow 2 \\ \searrow -1 \end{cases} \Rightarrow \alpha \begin{cases} \nearrow -1 \\ \searrow 2 \end{cases} \rightarrow$ 2 حالت

$4x^2+kx^2-9x-2=0 \rightarrow$ [illegible] $+9-2=0 \quad 9-9+k=0 \rightarrow k=-3$ ✓

$\alpha+\beta=2m \quad \alpha\beta=m \quad \sqrt{\alpha}-\sqrt{\beta}=1 \quad \alpha+\beta-2\sqrt{\alpha\beta}=1 \rightarrow 2m-2\sqrt{m}-1=0$

$\rightarrow m=1 \quad x^2-2mx+m=0 \rightarrow$ ↗

$2x^2-mx-m=0 \xrightarrow{m=1} 2x^2-x-1=0 \quad \alpha\beta=\frac{c}{a} \Rightarrow \frac{-1}{2}$ ✓



$S_{\triangle ABC} = \frac{3}{4}$ ، $y = 2x^2 - (m+2)x + m$

$2x^2 - (m+2)x + m$ $\qquad$ $\frac{m+2}{4}$ $\qquad$ $\frac{\sqrt{m^2+4m+4-8m}}{2} = \frac{\sqrt{(m-2)^2}}{2}$ (٢)

$\frac{\text{طول}}{\text{ارتفاع}} = \frac{\frac{m-2}{2} \times 2}{2} = \frac{3}{4}$ ← $\frac{m^2-2m}{4} = \frac{3}{4} \rightarrow m^2-2m = 3 \quad m^2-2m-3=0$

$y = x^2 + x + 1 \rightarrow \frac{-b}{2a} = \left(\frac{-1}{2}\right) = x_s$

$y = x^2 - 2x + 1 \Rightarrow x_s = \frac{2}{2}$

۴) ②

$y = ax^2 + 3x + a$ min $\Big|\ \frac{7}{8} = \frac{-\Delta}{4a}$ $\frac{-9 - 4a^2}{4a} \Rightarrow \frac{4a^2 - 9}{4a} = \frac{7}{8}$

$32a^2 - 72 = 28a \Rightarrow 32a^2 - 28a - 72 = 0 \rightarrow 8a^2 - 7a - 18 = 0$

$(a+9)(a-16) = 0$

$\rightarrow a = \frac{-9}{8} \rightarrow$ max دارد $\rightarrow$ کمترین مقدار $\frac{7}{8}$ نمی‌شود ✗

$a = \frac{16}{8} = 2$ ✓

۷) $S>0$ ، $\frac{S}{2}\in N \to \frac{a}{2}+\frac{1}{2}\in N$ $x^2-(a+1)x+a=0$

$-\frac{b}{a}>0$ $\frac{a+1}{1}>0$ $a+1>0$ $a>-1$ ، $\frac{a}{2}+\frac{1}{2}\in N$ → [illegible]

$\to a=\{x \mid x=2k-1, x\in N\}$ $x^2-2x+1=0 = (x-1)^2=0$

$-x^2-6x+27=0$ $(x-3)(x-1)$ → $x=3$ $x=1$

$x^2-(2a+1)x+b=0$ $x^2-10x+b=0$ $\Delta>0$ → $\frac{\Delta}{4}>0$

$a=0$ → با توجه معادله اول $P=\frac{c}{a}=\frac{b}{1}$ ، $\frac{\Delta}{4}=\frac{b}{4}$ → $\frac{b}{4}>0$

$\to b=\{x \mid x=2k, k\in N\}$ → $b=\{2,4,6,8,\dots\}$ مجموع دو عدد متوالی زوج = ۱۰

→ اعداد = ۴,۶ → $(x-6)(x-4)$ → $4\times 6=24$

→ $b=24$ اختلاف = $36-12=24$

۸) $y=-ax^2+ax+2$ $y=-a\left(\frac{1}{2}\right)^2+a\left(\frac{1}{2}\right)+2=-\frac{a}{4}+\frac{a}{2}+2=\frac{a}{4}+2$

$x_S=\frac{-b}{2a}=\frac{-a}{2(-a)}=\frac{1}{2}$ $\left(\frac{1}{2},\frac{a}{4}+2\right)$

$y=2bx^2-bx-1$ $x_S=\frac{-(-b)}{2(2b)}=\frac{b}{4b}=\frac{1}{4}$ $y=2b\left(\frac{1}{16}\right)-b\left(\frac{1}{4}\right)-1=\frac{-b}{8}-1$

$\left(\frac{1}{4},\frac{-b}{8}-1\right)$ $\left(x=\frac{1}{4}\right)$ → هم‌گذری → $y=2b\left(\frac{1}{4}\right)-b\left(\frac{1}{2}\right)-1=\frac{b}{2}-\frac{b}{2}-1=-1$

هم‌گذری $\frac{1}{4}$ در معادله اول → $y=-a\left(\frac{1}{16}\right)+a\left(\frac{1}{4}\right)+2=-\frac{a}{16}+\frac{a}{4}+2=\frac{3a}{4}-1=-\frac{1}{4}-\frac{b}{8}-1$

$b=-4$ $a=12$ $a-b=16$

۹) $\beta = 1$, (۲)

$-1 = \alpha, \beta$ $\quad y = 25ax^2 + 4x + \beta \quad \alpha + \beta = \frac{-4}{25a} \quad \alpha\beta = \frac{\beta}{25\alpha}$

$\beta = 25a\alpha\beta \quad \div\beta \rightarrow 25a\alpha = 1 \rightarrow a = \frac{1}{25\alpha}$

$\rightarrow x_S = \frac{4}{25 \times 2 \times \frac{1}{25\alpha}}$ $\quad x_S = -\frac{b}{2a} = \frac{-4}{2a} = -\frac{2}{5a}$ + فرضیات قبل

$x_S = -2\alpha$ ✓

در ناحیه ۱ قرار دارد.

$\alpha < \beta \rightarrow \alpha\times\beta = -\frac{\beta}{25\alpha}$

$\alpha < 0 \quad \beta > 0 \rightarrow -2\alpha > 0 \rightarrow x_S > 0$

$\alpha\times\beta - \frac{\beta}{25\alpha} = 0 \quad \beta\left(\alpha - \frac{1}{25\alpha}\right) = 0 \rightarrow \begin{cases} \beta = 0 \\ \alpha = \pm\frac{2}{10} \end{cases}$ $\rightarrow \alpha = -\frac{2}{10}$

۱۰) (۲)

$a + b = \frac{-b}{a} = a^2 + b^2 - 12 \quad ab = a + b - 1 \rightarrow a^2 + b^2 = (a+b)^2 - 2ab$

$\rightarrow a + b = (a+b)^2 - 2ab - 12 \xrightarrow{ab = a+b-1} \sout{a+b = (a+b)^2 - 2(a+b-1)}$

$a + b = (a+b)^2 - 2(a+b-1) - 12 \xrightarrow{a+b=S} S^2 - 2(S-1) - 12 = S \rightarrow S^2 - 3S - 10 = 0$

$= (S-5)(S+2) \quad a + b = S = S = 5$ ✓ چون a, b طبیعی اند