پاسا ضیائی — شماره تکلیف ۲۴ — دهم ریاضی A — ۱۹٫۷۵

Subject: Date

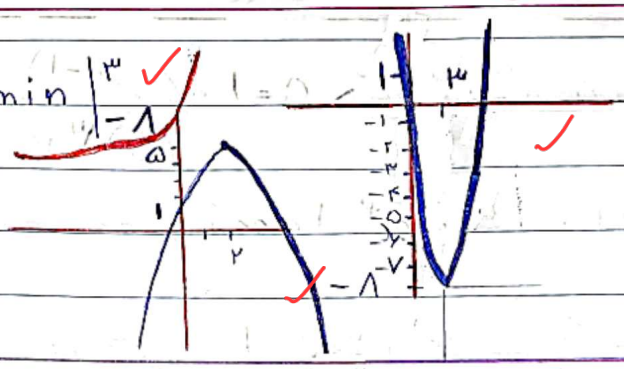
۱- الف) $y=2x^2-4x+1 \to \min \Big|_{-1}^{\frac{-b}{2a}=\frac{4}{4}=1} \quad 2(1)^2-4(1)+1=-1$ ✓

ب) $y=-2x^2+3x-5 \to \max \Big|_{\frac{-\Delta}{4a}=\frac{-31}{-8}=\frac{31}{8}}^{\frac{-b}{2a}=\frac{-3}{-4}=\frac{3}{4}}$ ✓ (۲)

۲- الف) $y=x^2-4x+1 \to \min \Big|_{-3}^{-3}$ ✓

ب) $y=-x^2+4x+1$ ✓

$\max \Big|_{0}^{4}$ ✓



(۲)

۳- $4x^3+kx^2-9x-2=0 \to 4(x-\alpha)(x-\beta)^2=4(x-\alpha)(x^2-2\beta x+\beta^2)$ (۲)

$\Rightarrow 4x^3-8\beta x^2+4\beta^2 x-4\alpha x^2+8\alpha\beta x-4\alpha\beta^2$

$=4x^3-(4\alpha+8\beta)x^2+(4\beta^2+8\alpha\beta)x-4\alpha\beta^2$

$\alpha+\beta=1 \xrightarrow{\beta=\frac{-1}{4}} \alpha-\frac{1}{4}=1 \Rightarrow \alpha=\frac{5}{4}$

$-4\alpha\beta^2=-2 \xrightarrow{\alpha\beta=-2} 8\beta=-2 \Rightarrow \beta=\frac{-1}{4}$

$k=-4\alpha-8\beta=-4\times\frac{5}{4}-8\times\frac{-1}{4}=-5+2=-3$ ✓

۴- $x^2-2mx+m=0 \to \sqrt{\alpha}-\sqrt{\beta}=1 \Rightarrow \underbrace{\alpha+\beta}_{2m}-2\sqrt{\underbrace{\alpha\beta}_{m}}=1$ (۲)

$\Rightarrow 2m-2\sqrt{m}=1 \to 2m-2\sqrt{m}-1=0 \Rightarrow \sqrt{m}=\frac{2\pm\sqrt{12}}{4}=\begin{cases}\frac{1+2}{2} & \checkmark \\ \frac{1-2}{2} & \times \to \sqrt{m}<0 \times\end{cases}$

$2x^2-mx-m \to \frac{c}{a}=\frac{-m}{1+2}=\frac{-(1)^2}{2}=\frac{-1}{2}$ ✓

۵- $2x^2-(m+2)x+m \to \Delta=m^2+4m+4-8m=m^2-4m+4=(m-2)^2$

$\frac{3}{4}=\frac{1}{2}\times\frac{\sqrt{\Delta}}{|a|}\times m=\frac{1}{2}\times\frac{|m-2|}{2}\times m$ (۲)

$\frac{m^2-2m}{4}=\frac{3}{4} \Rightarrow m^2-2m-3=0$ ✓

$\frac{-m^2+2m}{4}=\frac{3}{4} \Rightarrow m^2-2m+3=0$ ✗ ($\Delta<0$)

$m^2-2m-3=0 \to (m-3)(m+1)=0$ ($m=3$, $m=-1$)

$y<\begin{matrix}x^2+x+1, & \frac{-b}{2a}=\frac{-1}{2} \\ x^2-2x+1 \to & \frac{-b}{2a}=\frac{3}{2}\end{matrix}$ ✓

Subject: Date

$y = ax^2 + 3x + a \longrightarrow \frac{-\Delta}{4a} = \frac{V}{A} \Rightarrow -\frac{9 - 4a^2}{a} = \frac{V}{2}$ — ۱۷۵ — ۶

$\Rightarrow -7a = 18 - 8a^2 \Rightarrow 8a^2 - 7a - 18 = 0$ — $a = \frac{9}{8}$ غ ق ق

$a^2 - 7a - 144 = 0 \Rightarrow (a+9)(a-16) = 0$ — a باید مثبت باشه تا min داشته باشه!

$x^2 - (a+1)x + a = 0 \xrightarrow{a+b+c=0} \alpha = 1, \beta$ — $-1 \notin \mathbb{N}$ ✗ ; 3 ✓ — ۳ — ۷

$\Rightarrow (x-1)(x-3) = x^2 - 4x + 3$

$x^2 - (3a+1) + b = 0 \xrightarrow{a=3} x^2 - 10x + b' = 0$ $\Rightarrow a = 3$

$z(z+2) = \frac{c}{a} \Rightarrow z^2 + 2z = b$ $\quad z + z + 2 = \frac{-b}{a} \Rightarrow 2z + 2 = 10 \Rightarrow z = 4$

$z(z+2) - \alpha\beta \xrightarrow{z=4, \alpha=1, \beta=3} 4 \times 6 - 1 \times 3 = 21$ ✓

$y = -ax^2 + ax + 2 \rightarrow \frac{-b}{2a} = \frac{-a}{-2a} = \frac{1}{2}$ $\quad \frac{-\Delta}{4a} = \frac{a}{4} + 2$ — ۸ —

$y = 2bx^2 - bx - 1 \rightarrow \frac{-b}{2a} = \frac{b}{4b} = \frac{1}{4}$ $\quad \frac{-\Delta}{4a} = \frac{-b}{8} - 1$

$2b\left(\frac{1}{2}\right)^2 - b\left(\frac{1}{2}\right) - 1 = \frac{a}{4} + 2 \Rightarrow \frac{a}{4} = -3 \Rightarrow a = -12$ ✓

$+12\left(\frac{1}{4}\right)^2 - 12\left(\frac{1}{4}\right) + 2 = \frac{-b}{8} - 1 \Rightarrow \frac{3}{4} = \frac{-b}{8} \Rightarrow -b = -6$ ✓ $\quad b - a = (-6) - (-12) = 6$ ✓

$y = 25\alpha x^2 + 4x + \beta \rightarrow m(x-\alpha)(x-\beta) = mx^2 - m(\alpha+\beta)x + m\alpha\beta$ — ۹ —

$m = 25\alpha \quad -m(\alpha+\beta) = 4 \quad m\alpha\beta = \beta \xrightarrow{m=25\alpha} 25\alpha^2 = 1 \Rightarrow \alpha = \pm\frac{1}{5}$ — ۳ —

$-\frac{1}{5}(\pm\frac{1}{5} + \beta) = 4 \Rightarrow \pm\frac{1}{5} + \beta = -\frac{4}{5} \Rightarrow \beta$ — $+1$ ✓ ; $-1 \rightarrow \beta > \alpha \rightarrow -1 > -\frac{1}{5}$ ✗ — $\Rightarrow m = \pm 5$

چون $\frac{4}{-10} = \frac{-b}{2a} > 0$ و $25\alpha < 0$ پس راس سهمی در ربع اول است

$x^2 - (a^2 + b^2 - 12)x + a + b - 1 = 0$ — ۳ — ۱۵

$a + b = a^2 + b^2 - 12 \Rightarrow x^2 - (a^2+b^2-12)x + \underbrace{a+b-1}_{a^2+b^2-12} = x^2 - (a^2+b^2-12)x + a^2 + b^2 - 13$

$\xrightarrow{a+b+c=0} a = 1, b = \frac{a^2+b^2-13}{1}$

$a + b = a^2 + b^2 - 12 \xrightarrow{a=1} 1 + b = b^2 - 11 \Rightarrow b^2 - b - 12 = 0 \Rightarrow (b-4)(b+3) = 0$

$\Rightarrow b$ — $4 \rightarrow x^2 - 5 + 4 \Rightarrow a + b = 1 + 4 = 5$ ✓ ; $-3 \Rightarrow x^2 + 2 - 3$ ✗ $\rightarrow -3 \notin \mathbb{N}$