شماره تکلیف ۲۴ - دهم پسر A

پارسا ضیائی

Subject: Date

۱-

الف) $y=2x^2-4x+1 \to \min \left| \begin{array}{l} -\frac{b}{2a}=\frac{4}{4}=1 \to 2(1)^2-4(1)+1=-1 \\ -1 \end{array} \right.$

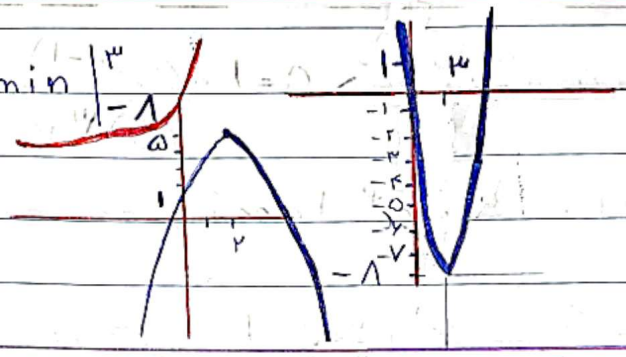
ب) $y=-2x^2+3x-5 \to \max \left| \begin{array}{l} -\frac{b}{2a}=\frac{-3}{-4}=\frac{3}{4} \\ \frac{-\Delta}{4a}=\frac{-31}{-8}=\frac{31}{8} \end{array} \right.$

۲-

الف) $y=x^2-6x+1 \to \min \left| \begin{array}{l} 3 \\ -8 \end{array} \right.$

ب) $y=-x^2+4x+1$

$\max \left| \begin{array}{l} 2 \\ 5 \end{array} \right.$



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$4x^3+kx^2-9x-2=0 \to 4(x-\alpha)(x-\beta)^2=4(x-\alpha)(x^2-2\beta x+\beta^2)$

$\Rightarrow 4x^3-8\beta x^2+4\beta^2 x-4\alpha x^2+8\alpha\beta x-4\alpha\beta^2$

$=4x^3-(4\alpha+8\beta)x^2+(4\beta^2+8\alpha\beta)x-4\alpha\beta^2$

$\alpha+\beta=1 \xrightarrow[\beta=\frac{-1}{4}]{} \alpha-\frac{1}{4}=1 \Rightarrow \alpha=\frac{5}{4}$

$-4\alpha\beta^2=-2 \xrightarrow{\alpha\beta=-2} 8\beta=-2 \Rightarrow \beta=\frac{-1}{4}$

$k=-4\alpha-8\beta=-\cancel{4}\times\frac{5}{\cancel{4}}-\overset{2}{\cancel{8}}\times\frac{-1}{\cancel{4}}=-5+2=-3$

۴-

$x^2-3mx+m=0 \to \sqrt{\alpha}-\sqrt{\beta}=1 \Rightarrow \underbrace{\alpha+\beta}_{3m}-2\underbrace{\sqrt{\alpha\beta}}_{m}=1$

$\Rightarrow 3m-2\sqrt{m}=1 \to$

$3m-2\sqrt{m}-1=0 \Rightarrow \sqrt{m}=\frac{2\pm\sqrt{16}}{6}=\begin{cases} \frac{1+2}{3} \checkmark \\ \frac{1-2}{3} \times \to \sqrt{m}<0 \times \end{cases}$

$2x^2-mx-m \to \frac{c}{a}=\frac{-m}{1+2}=\frac{-(1)^2}{2}=\frac{-1}{2}$

۵-

$2x^2-(m+2)x+m \to \Delta=m^2+4m+4-8m=m^2-4m+4$
$=(m-2)^2$

$\frac{3}{4}=\frac{1}{2}\times\frac{\sqrt{\Delta}}{|a|}\times m=\frac{1}{2}\times\frac{|m-2|}{2}\times m$

$\frac{m^2-2m}{4}=\frac{3}{4} \Rightarrow m^2-2m-3=0 \checkmark$

$\frac{-m^2+2m}{4}=\frac{3}{4} \Rightarrow m^2-2m+3=0 \times$
$\Delta<0$

$m^2-2m-3=0 \to \underbrace{(m-3)}_{\to 3}\underbrace{(m+1)}_{\to -1}=0$

$y \begin{cases} x^2+x+1 \to -\frac{b}{2a}=\frac{-1}{2} \\ x^2-3x+1 \to -\frac{b}{2a}=\frac{3}{2} \end{cases}$

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۶- $y = ax^2 + 3x + a \longrightarrow \frac{-\Delta}{4a} = \frac{7}{8} \Rightarrow -\frac{9 - 4a^2}{a} = \frac{7}{2}$

$\Rightarrow -7a = 18 - 8a^2 \Rightarrow 8a^2 - 7a - 18 = 0$ $\quad a = \begin{cases} -\frac{9}{8} \\ 2 \end{cases}$

$a^2 - 7a - 144 = 0 \Rightarrow (a+9)(a-16) = 0$

۷- $x^2 - (a+1)x + a = 0 \xrightarrow{a+b+c=0} \alpha = 1, \beta \begin{cases} -1 \times & -1 \notin \mathbb{N} \\ 3 \checkmark \end{cases}$

$\Rightarrow (x-1)(x-3) = x^2 - 4x + 3$

$\Rightarrow a = 3$

$x^2 - (\overset{3}{a} + 1) + b = 0 \xrightarrow{a=3} x^2 - 16x + b' = 0$

$z(z+2) = \frac{c}{a} \Rightarrow z^2 + 2z = b \qquad z + z + 2 = \frac{-b}{a} \Rightarrow 2z + 2 = 10$

$\Rightarrow z = 4$

$z(z+2) - \alpha\beta \xrightarrow{z=4, \alpha=1, \beta=3} 4 \times 6 - 1 \times 3 = 21$

۸- $y = -ax^2 + ax + 2 \rightarrow \frac{-b}{2a} = \frac{-a}{-2a} = \frac{1}{2} \qquad \frac{-\Delta}{4a} = \frac{a}{4} + 2$

$y = 2bx^2 - bx - 1 \rightarrow \frac{-b}{2a} = \frac{b}{4b} = \frac{1}{4} \qquad \frac{-\Delta}{4a} = \frac{-b}{8} - 1$

$2b\left(\frac{1}{2}\right)^2 - b\left(\frac{1}{2}\right) - 1 = \frac{a}{4} + 2 \Rightarrow \frac{a}{4} = -3 \Rightarrow a = -12$

$+12\left(\frac{1}{4}\right)^2 - 12\left(\frac{1}{4}\right) + 2 = \frac{-b}{8} - 1 \Rightarrow \frac{3}{4} = \frac{-b}{8} \Rightarrow -b = -6 \qquad b - a = (-6) - (-12) = 6$

۹- $y = 25\alpha x^2 + 4x + \beta \rightarrow m(x-\alpha)(x-\beta) = mx^2 - m(\alpha+\beta)x + m\alpha\beta$

$m = 25\alpha \quad -m(\alpha+\beta) = 4 \quad m\alpha\cancel{\beta} = \cancel{\beta} \xrightarrow{m=25\alpha} 25\alpha^2 = 1 \Rightarrow \alpha = \pm\frac{1}{5}$ ✓ غ ق ق

$\Rightarrow m = \pm 5$

$\mp 5\left(\pm\frac{1}{5} + \beta\right) = 4 \Rightarrow \pm\frac{1}{5} + \beta = \mp\frac{4}{5} \Rightarrow \beta \begin{cases} +1 \checkmark \\ -1 \rightarrow \beta > \alpha \rightarrow -1 > -\frac{1}{5} \times \end{cases}$

چون $\frac{-4}{-10} = \frac{-b}{2a} > 0$ و $25\alpha < 0$ پس رأس سهمی در ربع اول است

۱۰- $x^2 - (a^2 + b^2 - 12)x + a + b - 1 = 0$

$a + b = a^2 + b^2 - 12 \Rightarrow x^2 - (a^2 + b^2 - 12)x + \underbrace{a + b - 1}_{a^2+b^2-12} = x^2 - (a^2 + b^2 - 12)x + a^2 + b^2 - 13$

$\xrightarrow{a+b+c=0} a = 1, b = a^2 + b^2 - 13$

$a + b = a^2 + b^2 - 12 \xrightarrow{a=1} 1 + b = b^2 - 11 \Rightarrow b^2 - b - 12 = 0 \Rightarrow (b-4)(b+3) = 0$

$\Rightarrow b \begin{cases} 4 \rightarrow x^2 - 5 + 4 \Rightarrow a + b = 1 + 4 = 5 \\ -3 \Rightarrow x^2 + 2 - 3 \times \rightarrow -3 \notin \mathbb{N} \end{cases}$