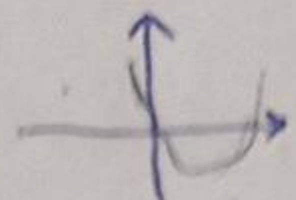
آیدین اشرفی - گروه A

۱)

الف) $y=3x^2-2x$ $\quad x(3x-2)=0$, $x_1=0$, $x_2=\frac{2}{3}$



$a>0$ $\quad -\frac{b}{2a}=\frac{1}{3}>0$

از ۳ می‌گذرد

ب) $y=-x^2+4x$ $\quad x(4-x)=0$, $x_1=0$, $x_2=4$

$a<0$ $\quad -\frac{b}{2a}=\frac{-4}{-2}=2$

از ۲ می‌گذرد.

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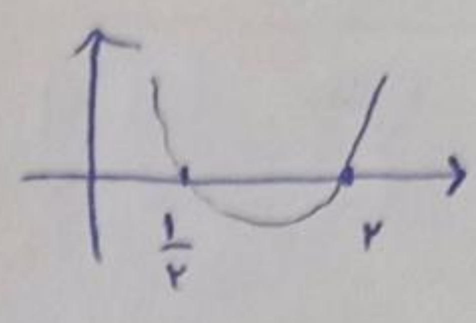
۲)

الف) $y=2x^2-5x+2$ $\quad (x-\frac{1}{2})(x-\frac{4}{2})=0$, $x_1=\frac{1}{2}$, $x_2=2$

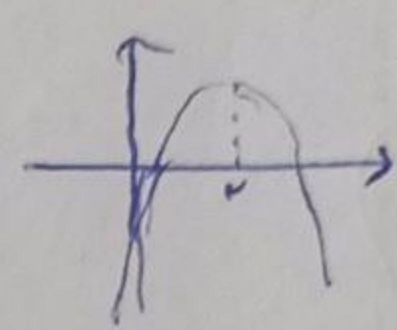


از ۱ و ۲ و ۴ می‌گذرد.

ب) $y=-x^2+4x-1$

$x=2$, $\quad \Delta=b^2-4ac=16-4=12>0$

$a<0$



از ۱ و ۳ و ۴ می‌گذرد

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۳)

$x^2-x-3=0$

الف) $\frac{\alpha+\beta}{\alpha-\beta}=\frac{\frac{-b}{a}}{\frac{\sqrt{\Delta}}{|a|}}=\frac{1}{\frac{\sqrt{13}}{1}}=\frac{\sqrt{13}}{13}$

$\Delta=1+12=13$

ب) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=S^2-2P=(-\frac{b}{a})^2-\frac{2c}{a}=1+2\times\frac{3}{1}=7$

ج) $\alpha^3+\beta^3=S^3-3PS=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=1-3(-3)(1)=1+9=10$

د) $\alpha^3-\beta^3=(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)=(\sqrt{13})^3+3(-3)\sqrt{13}=4\sqrt{13}$

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۴)

$x^2-ax+a \xrightarrow{\text{مماس}} (x-2)^2=x^2-4x+4 \Rightarrow a=4$

$\xrightarrow{\text{ثابت رد}}$ $\Delta<0$ $\quad a^2-4a<0$

0		4	
+	−		+

$\Rightarrow a\in(0,4]$

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۵)

$\alpha+\beta=\frac{12}{3}=4$ $\quad \alpha\beta=\frac{-a}{3}$ $\quad 2\alpha^2+\beta^2-4a=7 \xrightarrow{\beta=4-\alpha}$

$2\alpha^2+(4-\alpha)^2-4\alpha=7$ $\quad 2\alpha^2+16+\alpha^2-8\alpha-4\alpha=7$ $\quad 3\alpha^2-12\alpha+9=0$

$\alpha^2-4\alpha+3=0$ $\quad (\alpha-3)(\alpha-1)=0$ $\quad \alpha=1$ یا $\alpha=3$ ریشه بزرگتر $\quad \alpha\beta=3=\frac{-a}{3}$

$\Rightarrow a=-9$ $\quad \frac{a}{\text{ریشه بزرگتر}}=\frac{-9}{3}=-3$

(۶)

$7-2a \in \mathbb{N} \rightarrow 7-2a > 0 \quad 3{,}5 > a$

$a-2 > 0 \quad 3{,}5 > a > 2$

$a > 2 ,\quad a-2 \in \mathbb{N} \quad a \in \mathbb{N} \Rightarrow a = 3$

$S(5, 3) \quad \dfrac{7-2a+2a+3}{2} = b \Rightarrow \underline{b = 5}$

$y = a(x-5)^2 + 3 \xrightarrow[y=1]{x=1} 1 = a(1-5)^2 + 3 \quad a = -\dfrac{1}{8}$

$y = -\dfrac{1}{8}(x-5)^2 + 3 \Rightarrow c = -\dfrac{25}{8} + 3 = \dfrac{-1}{8}$

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(۷)

$\alpha + \beta = 1 \rightarrow \beta = 1 - \alpha \quad 2\beta^2 + \alpha^2 - \beta = \dfrac{17}{20} \quad (\beta^2 + \alpha^2) + (\beta^2 - \beta) = \dfrac{17}{20} \Rightarrow$

$(\beta + \alpha)^2 - 2\alpha\beta + \beta(\beta - 1) = \dfrac{17}{20}$

$1 - 2\alpha\beta + \beta(-\alpha) = \dfrac{17}{20}$

$1 - 3\alpha\beta = \dfrac{17}{20}$

$1 - 3\alpha(1-\alpha) = \dfrac{17}{20}$

$1 - 3\alpha + 3\alpha^2 = \dfrac{17}{20}$

$\alpha^2 - \alpha + \dfrac{1}{20} = 0 \quad |\alpha - \beta| = \dfrac{\sqrt{\Delta}}{|a|} = \dfrac{2\sqrt{5}}{1} = 2\sqrt{5}$

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(۸)

$x = \dfrac{-5+1}{2} = -2 \quad y = a(x+2)^2 - \dfrac{1}{2}$

$\dfrac{3}{2} = a(0+2)^2 - \dfrac{1}{2} \quad 4a - \dfrac{1}{2} = \dfrac{3}{2} \quad a = \dfrac{1}{2}$

$y = \dfrac{1}{2}(x+2)^2 - \dfrac{1}{2} \quad \beta = \dfrac{1}{2}(1+2)^2 - \dfrac{1}{2} \Rightarrow \boxed{\beta = 4}$

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(۹)

$x^2 + 6x + a = 0 \quad \alpha + \beta = \dfrac{-b}{a} = -6 \quad \alpha\beta = \dfrac{c}{a} = a$

$\begin{array}{ccccc} & \alpha & & \beta & 0 \\ + & | & - & | & + \end{array}$

$x = \dfrac{-6 \pm \sqrt{36 - 4a}}{2} \quad \left.\begin{array}{l} \alpha = -3 - \sqrt{9-a} \\ \beta = -3 + \sqrt{9-a} \end{array}\right\}$ جایگذاری در رابطه داده شده $\Rightarrow 3(-3-\sqrt{9-a})^2 + 2(-3+\sqrt{9-a})^2 = 12\sqrt{2} + 85$

تغییر متغیر $\sqrt{9-a} = t \quad 3(-3-t)^2 + 2(-3+t)^2 = 45 + 6t + 5t^2 = 12\sqrt{2} + 85$

$\boxed{a = 1} \Leftarrow 9 - a = 8 \Leftarrow \sqrt{9-a} = 2\sqrt{2} \Leftarrow \boxed{t = 2\sqrt{2}}$: از تناسب دو طرف تساوی داریم

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(۱۰)

$\dfrac{1}{\sqrt{\alpha}} + \dfrac{1}{\sqrt{\beta}} = 5 \quad \left(\dfrac{\sqrt{\beta} + \sqrt{\alpha}}{\sqrt{\alpha\beta}}\right)^2 = 25 \quad \dfrac{\alpha + \beta + 2\sqrt{\alpha\beta}}{\alpha \cdot \beta} = 25 \quad \dfrac{\frac{m+14}{36} + \frac{1}{3}}{\frac{1}{36}} = 25$

$\boxed{m = -1} \quad mx^2 + 3x + 2 = 0 \xrightarrow{m=-1} -x^2 + 3x + 2 = 0 \quad \boxed{\dfrac{c}{a} = x_1 x_2 = -2}$: حاصل ضرب ریشه‌ها