مهدی جان شکری — دفتر A — تکلیف شماره ۵

۱) الف) $y = 2x^2 - 2x$

ext | $\frac{1}{2}$ | $-\frac{1}{2}$

از نامساوی نمی‌گذرد.

ب) $y = -x^2 + 4x$

ext | ۲ | ۴

از نامساوی ۲ نمی‌گذرد.

۲) الف) $y = 2x^2 - 5x + 2$

ext | $\frac{5}{4}$ | $-\frac{9}{8}$

$(\frac{5}{4}, -\frac{9}{8})$

ب) $y = -x^2 + 4x - 1$

ex | ۲ | ۳

$(2, 3)$

۱۳) الف) $\frac{\alpha+\beta}{\alpha-\beta} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

$x^2 - x - 3 = 0$

$\alpha + \beta = 1 = \frac{-b}{a}$

$\alpha\beta = \frac{c}{a} = -3$

$|\alpha - \beta| = \sqrt{13}$

ب) $\alpha^2 + \beta^2 = S^2 - 2P = 1 - (-6) = 7$

ج) $\alpha^3 + \beta^3 = S^3 - 3SP = 1 - 3(-3) = 10$

د) $\alpha^5 - \beta^5 = (\alpha - \beta)(\alpha^4 + \beta^4 + \alpha\beta) \Rightarrow (\sqrt{13})(1 + 3) = 4\sqrt{13}$

۴) $y = (x - 2)(x^2 - ax + a)$

$x = 2 \Rightarrow$ یکی از ریشه‌ها $\Rightarrow x^2 - ax + a$ نباید ریشه مضاعف داشته باشد.

$\Rightarrow \Delta < 0 \Rightarrow a^2 - 4a < 0 \Rightarrow a(a-4) < 0$ — جدول: $+$ | 0 | $-$ | 0 | $+$ — $0 < a < 4$

$\Rightarrow a = 4 \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow (x-2)^2 = 0$ (ریشه مضاعف)

$\Rightarrow a = (0, 4) - \{4\}$

۵) $2x^2 - 12x + 4 - a = 0$

$\Rightarrow x^2 - 6x = \frac{a}{2} - 2$

$\Rightarrow \alpha^2 - 6\alpha = \frac{a}{2} - 2$

$\alpha^2 + \beta^2 = S^2 - 2P \Rightarrow 14 - \frac{3a}{2}$

$\alpha^2 - 6\alpha + \beta^2 - 7 = 0$

$\Rightarrow \frac{a}{2} + 14 - \frac{3a}{2} - 7 = 0$

$\Rightarrow 7 - \frac{a}{2} = 0 \Rightarrow 14 - a = 0 \Rightarrow a = 14$

$S = 6$

$P = \frac{a}{2}$

$\frac{14}{\sqrt{13}} = -6 + 2\sqrt{13}$

.14

$V-4a>0 \to c,d>a$
$\approx -2>0 \to a>2$
$\Big\} \xrightarrow{a\in N} a=3$

$B(1,1)$
$A(a,1)$
$\Big\} x=\frac{1+a}{2}=3=b \Big\}$ ext $\Big|\begin{matrix} d \\ c \end{matrix}$

$|y|=\left|-\frac{1}{\Lambda}(3)^2+2\right| \to |y|=\left|\frac{1}{\Lambda}\right|=\frac{1}{\Lambda}$

$y=a(x-h)^2+k$

$0=-\frac{1}{\Lambda}(x-d)^2+2$
$\to |d\pm 2\sqrt{4}|$
$=|d-2\sqrt{4}|$

$\to y=a(x-d)^2+2$

$1=4a+2 \to a=-\frac{1}{\Lambda}$

.15

$ax^2-ax-b=0 \to \alpha+\beta=1$
$\to \alpha\beta^2-\alpha\beta=b$
$\to \beta^2-\beta=\frac{b}{\alpha}$
$\to \alpha\beta=\frac{-b}{\alpha}$

$\alpha^2+\beta^2=1+\frac{2b}{\alpha}$

$=\frac{20b}{a}+20+\frac{10b}{a}=10 \to \frac{40b}{a}=-20 \to 20b=-a$

$\to -20bx^2+20bx-b=0 \Rightarrow -b(20x^2-20x+1)=0$

$\Rightarrow \frac{2\sqrt{5}}{-5}$

$\alpha-\beta=\frac{\sqrt{\Delta}}{|a|}=\frac{\sqrt{400-80}}{20}$

.16

$x=\frac{1-a}{2}=-2$
$y=a(x-h)^2+k \to y=a(x+2)-\frac{1}{2}$
$y=-\frac{1}{2}$
$\to \frac{2}{2}=a(2)^2-\frac{1}{2} \to a=\frac{1}{2}$
$\beta=\frac{1}{2}(2)^2-\frac{1}{2}=$ (1)

.17

$x^2+4x+a=0$
$\Rightarrow x^4=-4x-a$
$\to a^2=-4-a$
$\to a^2+4a+4=0 \to a=-2-2\sqrt{2}$

$\alpha+\beta=-4 \to \beta=(a+4)$
$\alpha\beta=a$

$2\alpha^2+2\beta^2=5\alpha^2+2(\alpha^2+\beta^2)=32-8a-4a-a=40a+12\sqrt{2}$

$12+12\sqrt{2}=-5a-4a=-5\alpha\beta-4\alpha=5\alpha^2+20\alpha$

$\Rightarrow \beta=-2+2\sqrt{2}$

$a=\alpha\beta=(-2-2\sqrt{2})(-2+2\sqrt{2})=$ (1)

.18

$\frac{1}{2\alpha}+\frac{1}{2\beta}=\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}}=3 \Rightarrow \frac{3}{4}=\sqrt{\alpha}+\sqrt{\beta} \to \frac{27}{16}=\alpha+\beta+\frac{2\sqrt{\alpha\beta}}{2}$

$\Rightarrow \alpha+\beta=\frac{12}{24}$
$24x^2-(m+14)x+1=0$
$\alpha\beta=\frac{1}{24}$
$\alpha+\beta=\frac{m+14}{24}$

$m+14=12 \Rightarrow m=-2$
$-x^2+2x+2=0 \to \alpha\beta=\frac{2}{-1}=$ (-2)