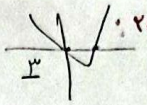
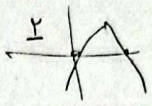
پارسا سماعی پسر دهم A تکلیف ۲۵:

(۱) از ناحیه‌ی سوم نمی‌گذرد

الف) $y=2x^2-2x$ $\quad x(2x-2)\to x_1=0,\ x_2=\frac{2}{2}$ $\quad c=0$



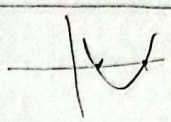
ب) $y=-x^2+4x$ $\quad x(-x+4)\to x_1=0,\ x_2=4$ $\quad c=0$



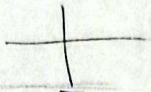
از ناحیه‌ی دوم نمی‌گذرد.

(۲) از نواحی اول، دوم و چهارم می‌گذرد.

الف) $y=2x^2-5x+2$ $\quad 2x^2-5x+2=0\Rightarrow x_1=+\frac{1}{2},\ x_2=+2$ $\quad a>0$



ب) $-x^2+4x-1=y$ $\quad \text{Max}\begin{cases}x=\frac{-b}{2a}=\frac{-4}{-2}=2\\ y_e=3\end{cases}$



از نواحی اول، سوم و چهارم می‌گذرد.

(۳) $y=x^2-x-3$ $\quad -\frac{b}{a}=\alpha+\beta=1$ $\quad \frac{c}{a}=\alpha\beta=-3$ $\quad |\alpha-\beta|=\frac{\sqrt{\Delta}}{|a|}=\sqrt{13}$

الف) $\frac{\alpha+\beta}{\alpha-\beta}=\frac{1}{\sqrt{13}}=\frac{\sqrt{13}}{13}$ $\quad$ ب) $\alpha^2+\beta^2=9-2\times-3=15=(\alpha+\beta)^2-2\alpha\beta$

ج) $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)=1\times(15-(-3)=18$ $\quad$ د) $\alpha^3-\beta^3=(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta)=\sqrt{13}(15-3)=12\sqrt{13}$

(۴) $y=(x-2)(x^2-ax+a)$ $\xrightarrow{\text{ریشه}}$ ریشه $=2\to x^2-ax+a=x^2-4x+4\to a=4$

سه ریشه $\Delta<0\to a^2-4a<0\Rightarrow a(a-4)<0$ $\quad a\in(0,4]$

(۵) $2x^2-12x-a=0\to a=2x^2-12x\Rightarrow x^2-4x=\frac{a}{2}\Rightarrow \alpha^2-4\alpha=\frac{a}{2}$

$2\alpha^2+\beta^2-4\alpha=7=\underbrace{\alpha^2-4\alpha}_{\frac{a}{2}}+\underbrace{\alpha^2+\beta^2}_{S^2-2P}=7\iff 14+a=7\Rightarrow a=-9$ $\quad \frac{-9}{2}=-3$

$2x^2-12x+9=0\Rightarrow x^2-4x+2=0\Rightarrow (x-1)(x-3)=0$ $\quad x=1,\ x=3$

(۶) $A(2a+3,a-2)$ $\quad a-2\geq 1$ $\quad a\geq 3$

$B(7-2a,a-2)$ $\quad 7-2a\geq 1$ $\quad 6\geq 2a$ $\quad 3\geq a$ $\Big\}\ a=3$ $\quad A(9,1)$ $\quad B(1,1)$ $\quad x_c=\frac{9+1}{2}=5$

$S(b,b-2)\Rightarrow S(5,3)$ $\quad y=a(x-5)^2+3$ $\xrightarrow{(1,1)}$ $1=a\times 16+3\Rightarrow a=-\frac{1}{8}$

$y=-\frac{1}{8}x^2-\frac{2\times 5}{8}+\frac{10}{8}x+3\Rightarrow c=-\frac{1}{8}$

(۷) $ax^2-ax-b=0\xrightarrow{\div a}x^2-x-\frac{b}{a}=0\Rightarrow \alpha+\beta=1,\ \alpha\beta=-\frac{b}{a}$ $\xrightarrow{x=\beta}\beta^2-\beta=\frac{b}{a}$

$4\beta^2+16\alpha^2-24\beta=17\Rightarrow 4\beta^2+4\alpha^2-6\beta=\frac{17}{4}\Rightarrow \beta^2+\alpha^2+\beta^2-\beta=\frac{17}{20}=\frac{a+10b}{a}\Rightarrow 17a=2a+40b\Rightarrow 15a=40b$

$+a=-2b$

$x^2-x+\frac{1}{20}=0$ $\quad (\alpha+\beta)^2-2\beta\alpha=1+\frac{2b}{a}$ $\quad \frac{b}{a}$

$|\alpha-\beta|=\frac{\sqrt{\Delta}}{|a|}=\frac{\sqrt{1-\frac{1}{5}}}{1}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$

(۸) $(-5,\beta),(1,\beta)$ $\quad x_e=\frac{-5+1}{2}=-2$ $\quad y=a(x+2)^2-\frac{1}{2}=ax^2+4a+4ax-\frac{1}{2}\xrightarrow[y=\frac{3}{2}]{x=0}4a-\frac{1}{2}=\frac{3}{2}$

$4a=2\Rightarrow a=\frac{1}{2}$

$\xrightarrow{(1,\beta)}\beta=\frac{1}{2}(1+2)^2-\frac{1}{2}=4$

(۹) $x^2+4x+a=0$ $\quad x=\frac{-4\pm 2\sqrt{9-a}}{2}$ $\quad \alpha=-2+\sqrt{9-a}\Rightarrow \alpha^2=13-a-4\sqrt{9-a}$

$\beta=-2-\sqrt{9-a}\Rightarrow \beta^2=13-a+4\sqrt{9-a}$

$2\alpha^2+2\beta^2=90-5a-4\sqrt{9-a}=\underbrace{12\sqrt{2}}_{4\times 2\sqrt{2}=4\sqrt{8}}+15\Rightarrow 5a+4\sqrt{9-a}=5+4\sqrt{8}\Rightarrow a=1$

(۱۰) $2mx^2-(m+12)x+1=0$ $\quad \alpha\beta=\frac{1}{2m}$ $\quad \frac{1}{\sqrt{\alpha}}+\frac{1}{\sqrt{\beta}}=\frac{\sqrt{\beta}+\sqrt{\alpha}}{\sqrt{\alpha\beta}}=5\Rightarrow \sqrt{\beta}+\sqrt{\alpha}=5\sqrt{\alpha\beta}\to \beta+\alpha+2\sqrt{\alpha\beta}=25\alpha\beta$

$\to \beta+\alpha+\frac{1}{2}=\frac{25}{2m}\Rightarrow \alpha+\beta=\frac{12}{2m}\to \frac{m+12}{2m}=\frac{12}{2m}\Rightarrow m=-1$

$mx^2+2x+2=0\Rightarrow -x^2+2x+2=0\Rightarrow \alpha'\beta'=\frac{c}{a}=-2$