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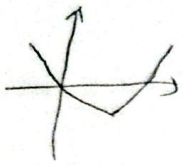
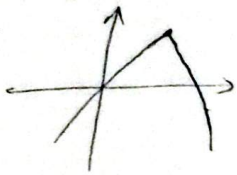
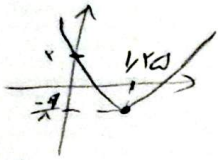
الف) سوم → سهمی min دهانه صعودی → محور تقارن نقطه $x=0$ باشد



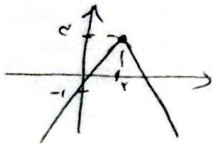
ب) دوم → سهمی max دهانه نزولی → محور تقارن $x=0$ نقطه



ج) اول و دوم و چهارم → min در $\frac{-9}{8}$ → $\frac{1}{2}$



د) اول و سوم و چهارم → max در $\frac{1}{c}$ →



ه) $\frac{1}{\sqrt{1+12}} = \frac{\sqrt{12}}{12}$

و) $S^2 - 2P = 1 - (2\alpha - c) = 7$

ز) $S^3 - 3SP = 1 - (3\alpha + \alpha - c) = 10$

ح) $(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta) = (\sqrt{12})(7-2) = 5\sqrt{12}$

$x^2 - ax + a = 0$ → $\alpha=\beta=2$ → $(x-2)^2 = x^2 - 4x + 4 \Rightarrow a = 4$

$\alpha<0$ → $x^2 - ax + a \to \Delta < 0 \Rightarrow a^2 - 4a < 0 \to$ ریشه‌ها $0, 4$ → $a \in (0,4)$

$x \in (0,4]$

$2\alpha^2 = 12\alpha + a \Rightarrow \alpha^2 = 6\alpha + \frac{a}{2} \Rightarrow 2\alpha^2 + \beta^2 - 6S = 7 \Rightarrow \underbrace{\alpha^2+\beta^2}_{14} + \frac{2a}{c} = 7$

$\Rightarrow a = -9 \to 2x^2 - 12x + 9 = 0 \Rightarrow x^2 - 6x + 2 = 0$ → $\frac{1}{c}$ → $\frac{a}{2} = -2$

$b = \frac{7 - 2\alpha + 2\alpha + c}{2} = 4 \Rightarrow b - 2 = c$

$k(x-4)^2 + 2 = ax - 4 \to a - 2 > 0 \Rightarrow a > 2$, $7 - 2a > 0 \Rightarrow a < 3.5$

$a = 3 \to k(3-4)^2 + 2 = 1 \to 14k + 2 = 1 \Rightarrow k = -\frac{1}{8}$

$x=0 \to -\frac{1}{8}(0-4)^2 + 2 = \frac{-2}{8} + 2 = -\frac{1}{8}$ نقطه $= \frac{1}{8}$

$|\alpha-\beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{(1)^2 - 4\alpha\beta}$

$\alpha\beta = 1 \begin{cases} \to \alpha = 1-\beta \to 4\beta^2 + 4(1-\beta)^2 - 4\beta = 17 \Rightarrow 4\beta^2 - 4\beta - 13 = 0 \\ \to \beta = \alpha - 1 \to 4(\alpha-1)^2 + 4\alpha^2 - 4(\alpha-1) = 17 \Rightarrow 4\alpha^2 - 4\alpha - 13 = 0 \end{cases}$

(۲) ضرب ریشه‌های α و β این معادله‌ها برابر $-\frac{13}{4}$ است

$\Rightarrow S = \frac{1}{4}$

$|\alpha-\beta| = \sqrt{1^2 - 4(\frac{1}{4})} = \sqrt{\frac{4}{4}} = \boxed{\frac{2\sqrt{4}}{4}}$ ✓

$x_v = -2, y_v = \frac{-1}{2} \Rightarrow a(x+2)^2 - \frac{1}{2} = y \xrightarrow{x=0} 4a - \frac{1}{2} = \frac{3}{2} \Rightarrow a = \frac{1}{2}$

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$\Rightarrow \frac{1}{2}(x+2)^2 - \frac{1}{2} = y \Rightarrow \beta = \frac{1}{2}(1+2)^2 - \frac{1}{2} = \boxed{4}$ ✓

$\alpha = \frac{-4 - \sqrt{64-4a}}{2} \Rightarrow \alpha^2 = \frac{64+64-4a+16\sqrt{64-4a}}{4} = 32 - a + 8\sqrt{64-4a}$

$2(\alpha^2+\beta^2) + \alpha^2 = 16\sqrt{2} + 48 \Rightarrow \alpha^2 = 16\sqrt{2} + 16 + 4a$

$2(\alpha^2+\beta^2) = 64-4a$

$64-4a$

$\Rightarrow 32 - a + 8\sqrt{64-4a} = 16 + 4a + 8(\sqrt{64})$

$\Rightarrow \sqrt{64-4a} = \sqrt{64} \Rightarrow a = 1$

$\Rightarrow 4x - 1 + 8\sqrt{64} = 16 + 4a \quad 4 + 8\sqrt{64} = 4$

$\Rightarrow \boxed{a = 1}$ ✓

$\frac{\sqrt{\alpha}}{\alpha} + \frac{\sqrt{\beta}}{\beta} = 0 \Rightarrow \frac{\beta\sqrt{\alpha} + \alpha\sqrt{\beta}}{\alpha\beta} = 0 \Rightarrow \sqrt{\alpha\beta}(\sqrt{\alpha} + \sqrt{\beta}) = 0$

$\Rightarrow \alpha\beta = 0$

$\Rightarrow \alpha + \beta + 2\sqrt{\alpha\beta} = 25\alpha\beta$

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$\Rightarrow \alpha\beta = \frac{16}{64} \Rightarrow \frac{m+14}{64} = \frac{16}{64} \Rightarrow m = -1$

$mx^2 + (m+2) = 0 \Rightarrow -x^2 + 2x + 2 = 0 \xrightarrow{\text{ضرب ریشه‌ها}} \frac{c}{a} = \frac{2}{-1} = \boxed{-2}$ ✓