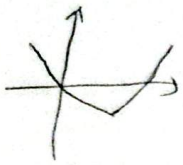
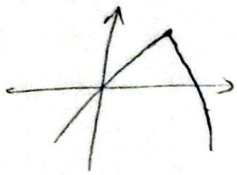
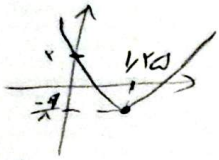
۱- الف) سوم → سهمی min دهان میباشد → محور تقارن نقطه $x=0$



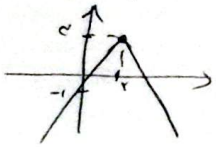
ب) دوم → سهمی max دهان میباشد → مثبت در نقطه $x=0$



۲- الف) اول و دوم و چهارم → $\min \left/ \begin{matrix} \frac{5}{4} \\ -\frac{9}{8} \end{matrix} \right.$



ب) اول و سوم و چهارم → $\max \left/ \begin{matrix} 2 \\ c \end{matrix} \right.$



۳- الف) $\frac{1}{\sqrt{1+12}} = \frac{\sqrt{12}}{12}$

ب) $S^2 - 2P = 1 - (2\alpha - c) = 7$

ج) $S^3 - 3SP = 1 - (3\alpha + 1\alpha - c) = 10$

د) $(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta) = (\sqrt{12})(7-2) = 5\sqrt{12}$

۴- $x^2 - ax + a = 0$

$\alpha=\beta=2$ → $(x-2)^2 = x^2 - 4x + 4 \Rightarrow \boxed{a=4}$

$\alpha<0$ → $x^2 - ax + a \to \Delta < 0 \Rightarrow a^2 - 4a < 0 \to \begin{matrix} 0 & 4 \\ - & + & - \end{matrix} \to a \in (0,4)$ → $x \in (0,4]$

۵- $2\alpha^2 = 12\alpha + a \Rightarrow \alpha^2 = 6\alpha + \frac{a}{2} \Rightarrow 2\alpha^2 + \beta^2 - 6\alpha = 7 \Rightarrow \underbrace{\alpha^2 + \beta^2}_{14} + 6\alpha + \frac{a}{2} = 7$

$\Rightarrow a = -9 \to 2x^2 - 12x + 9 = 0 \Rightarrow x^2 - 6x + 2 = 0$ → $\begin{matrix} 1 \\ c \end{matrix}$ → $\frac{a}{2} = \boxed{-3}$

$b = \frac{7 - 2a + 2a + c}{2} = 4 \Rightarrow b - 2 = c$

۶- $k(x-4)^2 + 2 = ax - 4 \to a - 2 > 0 \Rightarrow a > 2$, $7 - 2a > 0 \Rightarrow a < 3.5$ → $a=3 \to k(3-4)^2 + 2 = 1$

$\to 1 + 2 = 1 \Rightarrow k = -\frac{1}{8}$

$\xrightarrow{x=0} -\frac{1}{8}(0-4)^2 + 2 = \frac{-16}{8} + 2 = -\frac{1}{8} \xrightarrow{\text{قرینه}} 1 = \frac{1}{8} = \boxed{\frac{1}{8}}$

$|\alpha-\beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{(1)^2 - 4\alpha\beta}$

$\alpha\beta = 1 \begin{cases} \to \alpha = 1-\beta \to 4\beta^2 + 4(1-\beta)^2 - 4\beta = 17 \Rightarrow 4\beta^2 - 4\beta - 13 = 0 \\ \to \beta = \alpha - 1 \to 4(\alpha-1)^2 + 4\alpha^2 - 4(\alpha-1) = 17 \Rightarrow 4\alpha^2 - 4\alpha - 13 = 0 \end{cases}$ ضرایب β, α مشابه هستند

$4\alpha^2 - 4\alpha - 13$

$\Rightarrow S = \frac{1}{4}$

$|\alpha-\beta| = \sqrt{1^2 - 4(\frac{1}{4})} = \sqrt{\frac{4}{9}} = \boxed{\frac{2\sqrt{3}}{3}}$

$x_v = -2, y_v = \frac{-1}{2} \Rightarrow a(x+2)^2 - \frac{1}{2} = y \xrightarrow{x=0} 4a - \frac{1}{2} = \frac{3}{2} \Rightarrow a = \frac{1}{2}$

$\Rightarrow \frac{1}{2}(x+2)^2 - \frac{1}{2} = y \Rightarrow \beta = \frac{1}{2}(1+2)^2 - \frac{1}{2} = \boxed{4}$

$\alpha = \frac{-4 - \sqrt{64-4a}}{2} \Rightarrow \alpha^2 = \frac{64+64-4a+16\sqrt{64-4a}}{4} = 32 - a + 8\sqrt{64-4a}$

$2(\alpha^2+\beta^2) + \alpha^2 = 12\sqrt{2} + 48 \Rightarrow \alpha^2 = 12\sqrt{2} + 16 + 4a$

$2(\underbrace{\alpha^2+\beta^2}_{64-4a}) + \underbrace{\alpha^2}_{32-a+8\sqrt{64-4a}} = 12\sqrt{2} + 48$

$\Rightarrow 32 - a + 8\sqrt{64-4a} = 16 + 4a + 3(\sqrt{64})$

$\Rightarrow \sqrt{64-4a} = \sqrt{64} \Rightarrow a = 1$

$\Rightarrow \frac{4a}{a} - 1 + 4\sqrt{64} = 16 + 4 + 8\sqrt{64}$

$\Rightarrow \boxed{a = 1}$

$\frac{\sqrt{\alpha}}{\alpha} + \frac{\sqrt{\beta}}{\beta} = 3 \Rightarrow \frac{\beta\sqrt{\alpha} + \alpha\sqrt{\beta}}{\alpha\beta} = 3 \Rightarrow \sqrt{\alpha\beta}(\sqrt{\alpha} + \sqrt{\beta}) = 3\alpha\beta$

$\Rightarrow \alpha + \beta + 2\sqrt{\alpha\beta} = 4\alpha\beta$

$\Rightarrow \alpha + \beta = \frac{16}{64} \Rightarrow \frac{m+14}{64} = \frac{16}{64} \Rightarrow m = -1$

$mx^2 + (m+2) = 0 \Rightarrow -x^2 + 3x + 2 = 0 \xrightarrow{\text{ضرایب}} \frac{c}{a} = \frac{2}{-1} = \boxed{-2}$