14,5

Date: Sa Su Mo Tu We Th Fr Subject: آزادسازی

۱- $x^2 - ax + b \xrightarrow{x=1} 1 - a + b = 0 \to b - a = -1 \quad (I)$

$x^2 - ax + b \xrightarrow{x=2} 4 - 2a + b = 0 \to b - 2a = -4 \quad (II)$ (۲)

$(II) - (I) = b - 2a - (b - a) = -4 - (-1) \to -a = -3 \to a = 3$ ✓

$\to 1 - 3 + b = 0 \to b = 2$ ✓ $\to a + b = 3 + 2 = 5$ ✓

۲- $(x - 2x)^2$: $\xrightarrow{x=-1} -1 - 2x = 0 \to \boxed{x = -\frac{1}{2}}$ ✓

$(k-2)x + m - 1 = 0 \xrightarrow{x=2} (k-2)2 + m - 1 = 0 \to 2k - 4 + m - 1 = 0$ (۱)

$\to 2k + m = 5 \xrightarrow[k \in \mathbb{N}]{a<0} \boxed{k = 1}$ ✓ $\to 2 + m = 5 \to \boxed{m = 3}$

$\to \frac{m}{n} + k = \frac{3}{-\frac{1}{2}} + 1 = -6 + 1 = -5$

۳- $-\frac{1}{2}x^2 + 2x + 4 > \frac{5}{2} \xrightarrow{\times 2} -x^2 + 4x + 8 > 5$ (دقت ۱۰۰) (۰/۵)

$\to -x^2 + 4x + 3 > 0 \xrightarrow{\times(-1)} x^2 - 4x - 3 < 0 \to b' = \frac{b}{2} = \frac{-4}{2} = -2$

$\to \Delta' = b'^2 - ac = 4 - (1)(-3) = 7 \to x = \frac{-b' \pm \sqrt{\Delta'}}{a} = \frac{-(-2) \pm \sqrt{7}}{1}$

$\to \begin{cases} x = 2 + \sqrt{7} \\ x = 2 - \sqrt{7} \end{cases} \to$

x		$2-\sqrt{7}$		$2+\sqrt{7}$	
P	$+$	0	$-$	0	$+$

$\to b = 2 + \sqrt{7}$, $a = 2 - \sqrt{7}$

$\to b - a = 2 + \sqrt{7} - (2 - \sqrt{7}) = \boxed{2\sqrt{7}}$

۴- $f(x) = x^3 - 2x^2 - x + 2 = 0 \xrightarrow{\text{ریشه‌های گویا}} x = 1$ (۲)

① تقسیم هورنر:

	1	-2	-1	$+2$
1		1	-1	-2
	1	-1	-2	0

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$\rightarrow f(x) = (x-1)(x^2-2x-3)$

$x^2-2x-3=0 \xrightarrow{a+b=c} \begin{cases} x=-1 & \text{غ ق ق} \\ x=3 & \end{cases} \Big\} \xrightarrow{x>0} x=1, 3$

x		0		1		3	
P		⊙	$+$	0	$-$	0	$+$

$\rightarrow (a,b) = (1,3)$ ✓

$\xrightarrow{\text{ساده‌تر}} f(2) = (2-1)(4-4-3) = -3$ ✓

$x = \frac{1+3}{2} = 2$

$(a-1)x^2+(a-1)x+1 \;;\; \Delta < 0 \rightarrow b^2-4ac<0$ (۲) ـ۵

$\rightarrow (a-1)^2-4(1)(a-1) = a^2-2a+1-4a+4 < 0 \rightarrow a^2-6a+5<0$

$\rightarrow (a-1)(a-5)=0 \rightarrow \begin{cases} a=1 \\ a=5 \end{cases}$ (I)

$a-1<0 \rightarrow a<1$ (II)

$I \cap II \rightarrow a \in \emptyset$ ✓

$\frac{m(m^3+m)}{m-2} = \frac{m^2(m^2+1)}{m-2} \rightarrow \begin{cases} m=0 \\ m=2 \end{cases}$ (۲) ـ۶

x		0		2	
P	$-$	0	$-$	⊙	$+$

$\rightarrow \boxed{m>2}$ ✓

$\frac{(x^2-x-6)(x-1)^2}{(x^2+x+1)(2-x)^2} \leq 0 \rightarrow \begin{cases} x=3 \\ x=-2 \\ x=1 \\ x=2 \end{cases}$ ـ۷ (۲)

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x	-2	1	2	3
P	$+$	$-$	$-$	$+$

$\rightarrow x \in [-2,2) \cup [3,+\infty)$ ✓

۸- $\frac{3x^2-2x}{x^2+4} = 2 \rightarrow 3x^2-2x = 2x^2+8 \rightarrow x^2-2x-8=0$ (۲)

$\rightarrow (x-4)(x+2)=0 \rightarrow \begin{cases} x=4 \\ x=-2 \end{cases} \rightarrow (a,b)=(-2,4)$ ✓

$\rightarrow \begin{matrix} a=-2 \\ b=4 \end{matrix} \} \rightarrow b-a=4-(-2)=6$ ✓

۹- $-1 < \frac{3x^2-5x}{x+1} < 0 \xrightarrow{x>0} x(3x-5)=0 \rightarrow \begin{cases} x=0 \\ x=\frac{5}{3} \end{cases}$ ✓ $\boxed{\frac{5}{3}}$ وقتا!

x	0	$\frac{5}{3}$
P	$+$	$-$	$+$

$\rightarrow 0 < x < \frac{5}{3}$ (I) (۱)

$\frac{3x^2-5x}{x+1} > -1 \rightarrow \frac{3x^2-5x}{x+1}+1 > 0 \rightarrow \frac{3x^2-5x+x+1}{x+1} > 0$

$\rightarrow \frac{3x^2-4x+1}{x+1} > 0 \rightarrow \Delta = 16-4(1)(3) = -2 \rightarrow x \in \mathbb{R}$ (II)

$x+1>0$

$\boxed{x>-1}$

I ∧ II $\rightarrow$ $0 < x < \frac{5}{3}$ | $\frac{5}{3}$

۱۰- $\frac{x^2-10}{x} \leq 3 \rightarrow \frac{x^2-10}{x} - 3 \leq 0 \rightarrow \frac{x^2-3x-10}{x} \leq 0$ (۲)

$\rightarrow \frac{(x-5)(x+2)}{x} \leq 0 \rightarrow \begin{cases} x=0 \\ x=5 \\ x=-2 \end{cases}$

x	-2	0	5
P	$-$	$+$	$-$	$+$

$\rightarrow x \in (-\infty,-2] \cup (0,5]$ ✓

۲- $x=4$ ریشه عبارت $(k-2)x+m-1$ است:

$$(k-2)4+m-1=4k+m-9$$

ضریب x در عبارت $(k-2)x+m-1$ باید صفر باشد چون به ازای $x<4$ عبارت مثبت است

$$k-2<0 \rightarrow k<2 \xrightarrow{k \text{ طبیعی است}} k=1$$

$$4k+m-9=0 \xrightarrow{k=1} m=5$$

$$\frac{m}{n}+k=\boxed{-14}$$

۳-

$$-\frac{1}{2}x^2+2x+4>\frac{7}{2} \xrightarrow{\times(-2)} x^2-4x-12<-7$$

$$(x-5)(x+1)<0 \rightarrow \begin{array}{ccccc} & -1 & & 5 & \\ + & | & - & | & + \end{array} \quad -1<x<5$$

$$b-a=5-(-1)=\boxed{6}$$

۹-

$$\frac{3x^2-4x}{x+1}>-1 \rightarrow \frac{\overbrace{3x^2-3x+1}}{3x^2-4x+x+1}>0 \xrightarrow[\Delta<0 \text{ همواره مثبت}]{3x^2-3x+1} x+1>0$$

$$x>-1 \quad (1)$$

$$\frac{x(3x-4)}{x+1}<0 \rightarrow \begin{array}{ccccc} & -1 & & 0 & & \frac{4}{3} & \\ - & | & + & \not{0} & - & \not{0} & + \end{array}$$

$$x<-1 \text{ , } 0<x<\frac{4}{3} \quad (2)$$

$$(1) \cap (2) \rightarrow \boxed{0<x<\frac{4}{3}}$$