۱- $x^2 - ax + b \xrightarrow{x=1} 1 - a + b = 0 \rightarrow b - a = -1 \quad (I)$

$x^2 - ax + b \xrightarrow{x=3} 9 - 3a + b = 0 \rightarrow b - 3a = -9 \quad (II)$

$(II) - (I) = b - 3a - (b - a) = -9 - (-1) \rightarrow -2a = -8 \rightarrow a = 4$

$\rightarrow 1 - 4 + b = 0 \rightarrow b = 3 \rightarrow a + b = 4 + 3 = 7$

۲- $(x - 3x)^2 : \xrightarrow{x=-1} -1 - 3x = 0 \rightarrow \boxed{x = -\frac{1}{3}}$

$(k-2)x + m - 1 = 0 \xrightarrow{x=2} (k-2)2 + m - 1 = 0 \rightarrow 2k - 4 + m - 1 = 0$

$\rightarrow 2k + m = 5 \xrightarrow[k \in \mathbb{N}]{a < 0} \boxed{k = 1} \rightarrow 2 + m = 5 \rightarrow \boxed{m = 3}$

$\rightarrow \frac{m}{n} + k = \frac{3}{-\frac{1}{3}} + 1 = -9 + 1 = -8$

۳- $-\frac{1}{2}x^2 + 2x + 4 > \frac{7}{2} \xrightarrow{\times 2} -x^2 + 4x + 8 > 7$

$\rightarrow -x^2 + 4x - 1 > 0 \xrightarrow{\times(-1)} x^2 - 4x + 1 < 0 \rightarrow b' = \frac{b}{2} = \frac{-4}{2} = -2$

$\rightarrow \Delta' = b'^2 - ac = 4 - (1)(1) = 3 \rightarrow x = \frac{-b' \pm \sqrt{\Delta'}}{a} = \frac{-(-2) \pm \sqrt{3}}{1}$

$\rightarrow \begin{cases} x = 2 + \sqrt{3} \\ x = 2 - \sqrt{3} \end{cases} \rightarrow$

x	$2-\sqrt{3}$	$2+\sqrt{3}$
P	$+ \quad 0 \quad -$	$0 \quad +$

$\rightarrow b = 2 + \sqrt{3}, \quad a = 2 - \sqrt{3}$

$\rightarrow b - a = 2 + \sqrt{3} - (2 - \sqrt{3}) = \boxed{2\sqrt{3}}$

۴- $f(x) = x^3 - 2x^2 - x + 2 = 0 \xrightarrow{\text{ریشه‌های گویا}} x = 1$

تقسیم هورنر:

	1	-2	-1	$+2$
①	1	-1	-2	0

Date: Sa Su Mo Tu We Th Fr Subject:

$\rightarrow f(x) = (x-1)(x^2-2x-3)$

$x^2-2x-3=0 \xrightarrow{a+b=c} \begin{cases} x=-1 \text{ ق ق} \\ x=3 \end{cases} \xrightarrow{x>0} x=1, 3$

x		0		1		3	
P		⊙	$+$	0	$-$	0	$+$

$\rightarrow (a,b)=(1,3)$

$\xrightarrow[x=\frac{1+3}{2}=2]{\text{میانگین}} f(2)=(2-1)(4-4-3)=-3$

$(a-1)x^2+(a-1)x+1 : \Delta<0 \rightarrow b^2-4ac<0$ ـ۵

$\rightarrow (a-1)^2-4(1)(a-1) = a^2-2a+1-4a+4<0 \rightarrow a^2-6a+5<0$

$\rightarrow (a-1)(a-5)=0 \rightarrow \begin{cases} a=1 \\ a=5 \end{cases}$ (I)

$a-1<0 \rightarrow a<1$ (II)

$I \cap II \rightarrow a \in \emptyset$

$\frac{m(m^3+m)}{m-2} = \frac{m^2(m^2+1)}{m-2} \rightarrow \begin{cases} m=0 \\ m=2 \end{cases}$ ـ۶

x		0		2	
P		$-$	0	$-$	⊙ $+$

$\rightarrow \boxed{m>2}$

$\frac{(x^2-x-6)(x-1)^2}{(x^2+x+1)(2-x)^2} \leq 0 \rightarrow \begin{cases} x=3 \\ x=-2 \\ x=1 \\ x=2 \end{cases}$ ـ۷

Date: Sa Su Mo Tu We Th Fr Subject:

x	-2	1	2	3
P	$+$	$-$	$-$	$+$

$\longrightarrow x \in [-2, 2) \cup [3, +\infty)$

$\frac{3x^2 - 2x}{x^2 + 4} = 2 \longrightarrow 3x^2 - 2x = 2x^2 + 8 \longrightarrow x^2 - 2x - 8 = 0$ ـ۸

$\longrightarrow (x-4)(x+2) = 0 \longrightarrow \begin{cases} x = 4 \\ x = -2 \end{cases} \longrightarrow (a, b) = (-2, 4)$

$\longrightarrow \begin{matrix} a = -2 \\ b = 4 \end{matrix} \Big\} \longrightarrow b - a = 4 - (-2) = 6$

$-1 < \frac{3x^2 - 6x}{x+1} < 0 \xrightarrow{x > 0} x(3x - 6) = 0 \longrightarrow \begin{cases} x = 0 \\ x = \frac{6}{3} \end{cases}$ ـ۹

x	0	$\frac{6}{3}$
P	$+$	$-$	$+$

$\longrightarrow 0 < x < \frac{6}{3}$ (I)

$\frac{3x^2 - 6x}{x+1} > -1 \longrightarrow \frac{3x^2 - 6x + 1}{x+1} > 0 \longrightarrow \frac{3x^2 - 6x + x + 1}{x+1} > 0$

$\longrightarrow \frac{3x^2 - 3x + 1}{x+1} > 0 \longrightarrow \Delta = 9 - 4(1)(3) = -3 \longrightarrow x \in \mathbb{R}$ (II)

$I \cap II \longrightarrow \boxed{0 < x < \frac{6}{3}}$

$\frac{x^2 - 10}{x} \leq 3 \longrightarrow \frac{x^2 - 10}{x} - 3 \leq 0 \longrightarrow \frac{x^2 - 3x - 10}{x} \leq 0$ ـ۱۰

$\longrightarrow \frac{(x-5)(x+2)}{x} \leq 0 \longrightarrow \begin{cases} x = 0 \\ x = 5 \\ x = -2 \end{cases}$

x	-2	0	5
P	$-$	$+$	$-$	$+$

$\longrightarrow x \in (-\infty, -2] \cup (0, 5]$