۱- الف) $\begin{cases} 3x - y = 9 \xrightarrow{\times 2} 6x - 2y = 18 \\ x + 2y = -4 \longrightarrow x + 2y = -4 \end{cases} \xrightarrow{+} 7x = 14 \Rightarrow x = 2 \Rightarrow y = -3 \Rightarrow \frac{x}{y} = \boxed{\frac{-2}{3}}$

ب) $\begin{cases} \frac{1}{x} - \frac{1}{y} = -1 \Rightarrow y - x = -xy \xrightarrow{\times(-3)} -3y + 3x = 3xy \\ \frac{3}{x} - \frac{7}{y} = -2 \Rightarrow 3y - 7x = -2xy \longrightarrow 3y - 7x = -2xy \end{cases}$

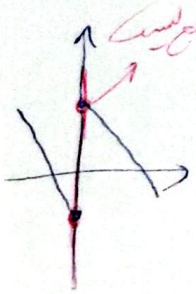
$4y - 14x = 0 \Rightarrow y = 2x \Rightarrow \frac{x}{y} = \frac{x}{2x} = \boxed{\frac{1}{2}}$

۲- $a \times 1 = -2 \Rightarrow a = -2$

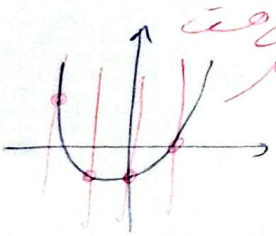
$f(a) + 2f(2) = 3f(1) \Rightarrow 2a + 4b = 3a + 3 \Rightarrow 4b = a + 3 \Rightarrow 4b = 0 \Rightarrow b = 0$

۳- $m^2 - 3m = -2 \Rightarrow m^2 - 3m + 2 = 0 \begin{cases} \boxed{m = 1} \\ m = 2 \longrightarrow \text{غ ق} \rightarrow m + 1 = 3 \rightarrow \text{عدد اول} \end{cases}$

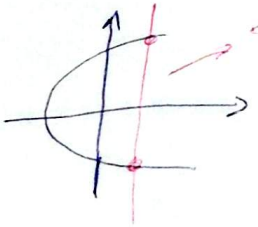
۴- الف)



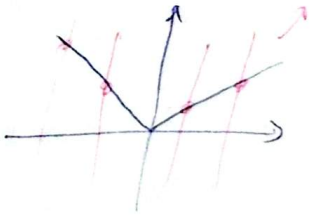
ب)



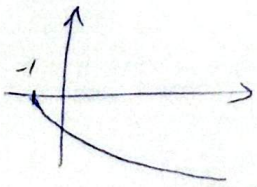
ج)



د)



۵- الف) $y = -\sqrt{x+1} \longrightarrow$ تابع است $\longrightarrow$



ب) $x = \frac{y}{\sqrt{1-y^2}} \longrightarrow x_1 = x_2$

$\Rightarrow \frac{y_1}{\sqrt{1-y_1^2}} = \frac{y_2}{\sqrt{1-y_2^2}} \xrightarrow{\text{به توان ۲}} \frac{y_1^2}{1-y_1^2} = \frac{y_2^2}{1-y_2^2} \xrightarrow{\text{طرفین وسطین}} y_1^2 = y_2^2$

$\Rightarrow y_1 = \pm y_2$

$\Rightarrow y_1 = y_2$

تابع است

y_1 و y_2 هم علامت $\leftarrow$ $\pm$

۶- $|y|=x \xrightarrow[x=1]{\text{مثلا}} |y|=1 \Rightarrow y=\pm 1 \longrightarrow$ تابع نیست

$(y+1)^2=-(x^2+x) \longrightarrow x_1^2+x_1=x_2^2+x_2$

$\Rightarrow (y_1+1)^2=(y_2+1)^2 \Rightarrow y_1+1=y_2+1 \Rightarrow y_1=y_2 \rightarrow$ تابع یک‌به‌یک

۷- $f_{(\sqrt{2}-1)}=\dfrac{(x+1)^2+1}{(x+1)^2+2}=\dfrac{(\sqrt{2}-1+1)^2+1}{(\sqrt{2}-1+1)^2+2}=\dfrac{4}{4}=\boxed{1}$

۸-

$y=2x-a \Rightarrow -4=-2-a \Rightarrow a=1$

$y=x^2+x+b \Rightarrow -2=-1-1+b \Rightarrow b=-2$

$2x-1=x^2+x-2 \Rightarrow x^2-x-1=0$ — چون به ازای $x=1$ مخرج صفر است، این ریشه قابل قبول نیست

$x^3-2x-1 \;\big|\; x-1 \;\big|\; x^2+x+1$

$-x^3-x^2$

$-x^2-2x$

$+x^2+x$

$-x-1$

$+x+1$

0

$\Rightarrow (x-1)(x^2+x+1)=0$ — چون x باید مثبت باشد $\Rightarrow x=1$

$\dfrac{-b}{a}=\dfrac{+1}{1}=\boxed{1}$

۹- $a+b=2a \Rightarrow b=a$

$a-2a+1=2a \Rightarrow -a+1=2a \Rightarrow 1=3a \Rightarrow \boxed{a=\frac{1}{3}}$

۱۰- تابع تابع همواره نسبت به اول باشد این یعنی

$f_{(x)}=\dfrac{4x^2-ax+c+1}{bx+c}=x \Rightarrow 4x^2-ax+c+1=bx^2+cx \Rightarrow b=4$

$\Rightarrow a=-c$

$\Rightarrow c=-1$

$\Rightarrow a+b+c=\boxed{6}$