

۱۸، ۷۵

الف) $3x - y = 4$
 $-3(x + 2y) = (-4) - 3$ $x + (-9)z = -4 \Rightarrow x = 2$

$-7y = 21 \Rightarrow y = -3$ $\frac{x}{y} = -\frac{2}{3}$ ✓ جواب نهایی

ب) $a = \frac{1}{x}, b = \frac{1}{y}$ $a - b = -1, \Delta a - \nabla b = -3$

$\nabla \times (a - b) = \nabla \times (-1) \Rightarrow \nabla a - \nabla b = -\nabla$

$(\Delta a - \nabla b) - (\nabla a - \nabla b) = -3 - (-1) \Rightarrow -\nabla a = 2 \Rightarrow a = -2 \Rightarrow -2 - b = -1 \Rightarrow b = -1$
 $\frac{1}{x} = -2 \Rightarrow x = -0.5, b = \frac{1}{y} = -1 \Rightarrow y = -1$ ✓ جواب نهایی

(۲)

۱

$f = \{(a, 2a), (1, a+1), (1, -2), (2, b)\} \Rightarrow a + 1 = -2$
 $\Rightarrow a = -3$ ✓

$f(a) + 2f(2) = 3f(1)$

$2a + b = 3a + 3$

$-4 + b = -4 \Rightarrow b = 0$ ✓ جواب نهایی

(۲)

۲

$f = \{(-1, m^2 - 3m), (3, 5), (-1, -2), (m+1, 9), (2, 4), (m^2 + 2, 2m + 1)\}$

$m^2 - 3m = -2 \Rightarrow m^2 - 3m + 2 = 0 \Rightarrow (m-2)(m-1) = 0 \Rightarrow m = 2$
 $m = 1$

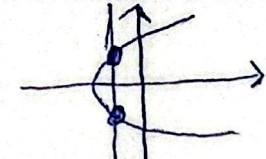
$m = 1 \Rightarrow \{(-1, -2), (3, 5), (-1, -2), (2, 9), (2, 4), (3, 5)\}$ تابع X

$m = 2 \Rightarrow \{(-1, -2), (3, 5), (-1, -2), (3, 9), (2, 4), (9, 9)\}$ تابع X

به ازای هیچ مقدار صحیح m تابع نمی شود. ✓ جواب نهایی

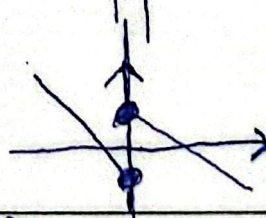
(۲)

ب) تابع نیست ✓



الف) تابع نیست ✓

د) تابع هست ✓



ج) تابع نیست ✓
 ه) تابع هست ✓

(۲)

۴

الف) $y = -\sqrt{x+1}, x+1 \geq 0 \Rightarrow x \geq -1$ ✓ تابع هست

ب) $x = \frac{y}{\sqrt{1-y^2}}, 1-y^2 > 0 \Rightarrow -1 < y < 1$ ✓ تابع هست

راه حل ؟!

(۱)

۵

الف) $|y| = x$ تابع نیست ✓ $x=2 \rightarrow y=2$
 $\rightarrow y=2 \rightarrow y=2$ مثال نقض

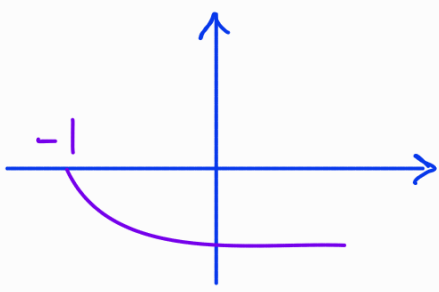
ب) $y^3 + 3y^2 + 3y + x^3 + x = 0$
 $(y+1)^3 = y^3 + 3y^2(1) + 3y(1)^2 + 1^3 = y^3 + 3y^2 + 3y + 1$
 $y^3 + 3y^2 + 3y = (y+1)^3 - 1$
 $((y+1)^3 - 1) + x^3 + x = 0$
 $(y+1)^3 - 1 + x^3 + x = 0 \rightarrow (y+1)^3 = 1 - x^3 - x \Rightarrow y = \sqrt[3]{1-x^3-x} - 1$

$f(x) = \frac{x^2+4x+5}{x^2+4x+7} \Rightarrow y = x^2 + 4x \Rightarrow f(x) = \frac{y+5}{y+7}$
 $x^2 = (\sqrt{3}-1)^2 = (\sqrt{3})^2 - 2(\sqrt{3})(1) + (1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$
 $4x = 4(\sqrt{3}-1) = 4\sqrt{3} - 4$
 $y = (4 - 2\sqrt{3}) + (4\sqrt{3} - 4) \Rightarrow y = 4 - 2\sqrt{3} + 4\sqrt{3} - 4 = 2\sqrt{3}$
 $= \frac{-1+5}{-1+7} = \frac{4}{6} = \frac{2}{3}$ جواب نای ✓

~~$f(x) = x^3 + ax + b$~~ $f(x) = x^3 + ax + b \Rightarrow f(x) = -2, x = -1 \Rightarrow -2 = -1 - a + b$
 $\Rightarrow -3 = -a + b$
 $y - 3x + a = 0 \rightarrow y = -2, x = 1 \rightarrow -2 - 3(-1) + a = 0 \rightarrow -2 + 3 + a = 0$
 $-1 + a = 0 \Rightarrow a = 1$
 $-3 = -(1) + b \Rightarrow b = -2$
 $x_1 = -1, x_2 = \frac{1-\sqrt{5}}{2}$
 $x_3 = \frac{1+\sqrt{5}}{2}$
 $x_2 + x_3 = \frac{1-\sqrt{5}}{2} + \frac{1+\sqrt{5}}{2} = 1$ 1 2 1, \sqrt{5} جواب نای ✓

$f = \{(r, a+b), (1, 2a), (-1, a-2b+1)\}$
 $a+b = 2a \rightarrow b = a$
 $a-2b+1 \Rightarrow a-2a+1 = 2a \Rightarrow 1 = 3a \Rightarrow a = \frac{1}{3}$ 2 جواب نای ✓

$x = \frac{fx^2 - ax + c + 1}{bx + 3} \Rightarrow x(bx + 3) = fx^2 - ax + c + 1 \Rightarrow bx^2 + 3x = fx^2 - ax + c + 1$
 $fx^2 - bx^2 - ax - 3x + c + 1 = 0 \Rightarrow (f-b)x^2 + (-a-3)x + (c+1) = 0$
 $f-b=0 \Rightarrow b=f$ $-a-3=0 \Rightarrow -a=3 \Rightarrow a=-3$
 $c+1=0 \Rightarrow c=-1$
 $a+b+c = f + (-3) + (-1) = 0$ 2 جواب نای ✓



تابع هست!

۵ ب

مخرج لسطرافبت اندیسن
 y_1 و y_2 هم علاقتند!

$$n = \frac{y_1}{\sqrt{1-y_1^2}}$$

$$n = \frac{y_2}{\sqrt{1-y_2^2}}$$

$$\Rightarrow \frac{y_1}{\sqrt{1-y_1^2}} = \frac{y_2}{\sqrt{1-y_2^2}} \xrightarrow[\text{طرفین وسطین}]{\text{توان ۲}} y_1^2 - y_1^2 y_2^2 = y_2^2 - y_1^2 y_2^2$$

$$y_1^2 = y_2^2$$

$$\xrightarrow[\text{تابع هست!}]{y_1 \text{ و } y_2 \text{ هم علاقتند}} \boxed{y_1 = y_2}$$