

if $x = a \Rightarrow f(a) = \begin{cases} a^2 + 2a \\ a^2 - 4 \end{cases} \Rightarrow a^2 + 2a = a^2 - 4 \Rightarrow 2a = -4 \Rightarrow a = -2$

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$g(x) = 2x + b \xrightarrow{x=2} 3 = f + b \Rightarrow b = -1 \Rightarrow f(x) = \frac{x^2 + a}{2x + 1} \xrightarrow{x=2} 3$
 $3 = \frac{f + a}{2} \Rightarrow f + a = 10 \Rightarrow a = 11 \Rightarrow f(x) = \frac{x^2 + 11}{2x + 1} \rightarrow f(1) = \frac{1 + 11}{2 + 1} = \frac{12}{3}$
 $= \boxed{4}$

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$D_f: \mathbb{R} - \{-1, 4\} \Rightarrow x = -1, x = 4$ *ریشه‌های است* $\Rightarrow 2x^2 + ax + b = 0$
 $\begin{cases} x = -1 \rightarrow 2 - a + b = 0 \\ x = 4 \rightarrow 32 + 4a + b = 0 \end{cases} \Rightarrow a = -4, b = -1$
 $\Rightarrow f(x) = \frac{4x + 1}{2x^2 - 4x - 1} \Rightarrow f(1) = \frac{4 + 1}{2 - 4 - 1} = \boxed{-\frac{5}{12}}$

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$D_f = \mathbb{R} - \{-1\} \rightarrow x = -1$ *ریشه یکتا* $\Rightarrow (-1)^2 = 1$ $\frac{-f + ax + b}{-f} = 1$
 $\Rightarrow b = -f$
 $\Rightarrow -fx^2 + ax - f \xrightarrow{x=-1} -f - a - f = 0 \Rightarrow a = -1 \Rightarrow f(x) = \frac{x^2 - \sqrt{3}}{-fx^2 - 1x - f}$
 $\Rightarrow a + b = -1 + (-f) = \boxed{-12}$

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$D_f = \mathbb{R} - \{1\}, f(x) = \frac{2x}{(x-1)(x^2 + mx + 1)} \Rightarrow x^2 + mx + 1$ *ریشه ندارد*
 $\Delta < 0 \Rightarrow m^2 - 4 < 0 \Rightarrow -2 < m < 2 \Rightarrow m \in (-2, 2)$ *ریشه‌ها ندارد*
 $\Rightarrow x = 1, y = 0 \Rightarrow 1 + m + 1 = 0 \Rightarrow m = -2 \Rightarrow \left\{ \begin{matrix} x = 1 \\ m = -2 \end{matrix} \right\}$
 $m: (-2, 2) \cup \{-2\} = \boxed{[-2, 2]}$

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$$f(x) = \sqrt{x - \frac{1}{x^2}} \Rightarrow x \neq 0, x - \frac{1}{x^2} \geq 0$$

$$\Rightarrow D_f = \left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, +\infty\right)$$

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$$mx^2 + 2mx + 1 \geq 0 \Rightarrow a = m \geq 0, \Delta \leq 0$$

$\Rightarrow f(m) = \sqrt{m} \geq 0 \Rightarrow D = \mathbb{R}$

$$\Rightarrow fm^2 - fm \leq 0 \Rightarrow fm(m-1) \leq 0$$

$$\Rightarrow m = [0, +\infty) \cap [0, 1] = [0, 1]$$

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$$f(x) = \begin{cases} \frac{fx^2 - 1}{2x - 1} ; x \neq \frac{1}{2} \\ g(x) = 2x + 1 ; x = \frac{1}{2} \end{cases} \Rightarrow a = \frac{1}{2} \Rightarrow 2a - 1 = 0 \Rightarrow a = \frac{1}{2}$$

$$\left. \begin{matrix} f \cdot \frac{1}{2} + k = 2 \cdot \frac{1}{2} + 1 \\ \Rightarrow 2 + k = 2 \Rightarrow k = 0 \end{matrix} \right\} \Rightarrow a + k = \frac{1}{2} + 0 = \frac{1}{2}$$

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$$f(x) = \begin{cases} \frac{ax^2 - \varepsilon}{2x + 2} ; x \neq -\frac{2}{\varepsilon} \\ 2ax + 2 ; x = -\frac{2}{\varepsilon} \end{cases} = g(x) = 2ax + b \Rightarrow x = 0 \Rightarrow \frac{0 - \varepsilon}{0 + 2} = 0 + b \Rightarrow b = -\frac{\varepsilon}{2}$$

$$x = -\frac{2}{\varepsilon} \rightarrow 2x - \frac{2}{\varepsilon} \cdot 2a + 2 = 2x - \frac{2}{\varepsilon} - 2 \Rightarrow g(x) = 2ax - 2$$

$$\Rightarrow -2a + 2 = -\varepsilon \Rightarrow -2a = -\varepsilon - 2 = a = \frac{\varepsilon + 2}{2} \Rightarrow a - b = \frac{\varepsilon + 2}{2} - \left(-\frac{\varepsilon}{2}\right) = \varepsilon + 1$$

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$$g(x) = x + 2 \neq f(x) = \begin{cases} \frac{x^2 - \varepsilon}{x - 2} ; x \neq 2 \\ 2a^2 + ax ; x = 2 \end{cases}$$

$$f = 2a^2 + 2a \Rightarrow a^2 + a - 2 = 0 \Rightarrow a = 1 \text{ or } -2$$

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