

۱۱. (۲)  $a^2 + \sqrt{a} = a^2 - \sqrt{a} \Rightarrow \sqrt{a} = -\sqrt{a}$   
 $\Rightarrow a = (-2) \checkmark$

(۲) (۲)  $f(x) = \frac{x^2 + a}{x - b} \xrightarrow{(2, 2)} \frac{x^2 + a}{x - 2} \Rightarrow f + a = 1 \Rightarrow -2a = 1 \Rightarrow a = -\frac{1}{2}$

$g(x) = \sqrt{x} + b \xrightarrow{(2, 2)} \sqrt{2} = \sqrt{2} + b \Rightarrow b = -1 \checkmark$

$f(x) = \frac{x^2 + 11}{x^2 + 1} \rightarrow f(1) = \frac{12}{2} = 6 \checkmark$

(۲) (۲)  $f(x) = \frac{f(x) + 1}{x^2 + ax + b}$

$DF = \mathbb{R} - \{-1, 1\} \xrightarrow{2^3 \frac{2}{1}} \sqrt{(x+1)(x-1)} = x^2 - 4x - 1 \checkmark$

$f(x) = \frac{f(x) + 1}{x^2 - 4x - 1} \rightarrow f(1) = \frac{2}{-15} \checkmark$

(۲) (۲)  $f(x) = \frac{x^2 - \sqrt{x}}{-x^2 + ax + b}$

$DF = \mathbb{R} - \{-1\} \xrightarrow{2^3 \frac{2}{1}} -\sqrt{(x+1)^2} = -x^2 - 2x - 1 \Rightarrow a = -2, b = -1$   
 $a + b = -1 - 2 = -3 \checkmark$

$$f(x) = \frac{rx}{(x-1)(x^2+mx+1)}$$

$$Df = \mathbb{R} - \{1\} \xrightarrow{\text{DZ}} \Rightarrow$$

$$x^2+mx+1 = (x-1)^2 \Rightarrow m = -2$$

$$[-2, 2] \quad \checkmark \quad \cdot (2)$$

$$\frac{-2 \quad 2}{+ \quad - \quad - \quad +}$$

$$\Delta < 0 \Rightarrow m^2 - 4 < 0 \rightarrow (m+2)(m-2) < 0 \Rightarrow m < 2$$

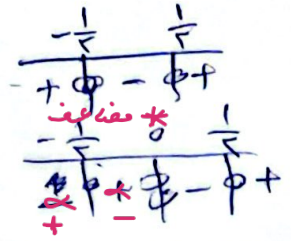
$$f(x) = \sqrt{x - \frac{1}{x^2}}$$

(4)  $\checkmark$

$$1) x \neq 0 \wedge x - \frac{1}{x^2} \geq 0 \Rightarrow \frac{x^3 - 1}{x^2} \geq 0$$

$$f(x^3 - 1) = 0 \Rightarrow x^3 - 1 = 0 \Rightarrow x^3 = 1 \Rightarrow x = \frac{1}{\sqrt[3]{1}} \Rightarrow x = 1 \Rightarrow \left\{ \frac{1}{\sqrt[3]{1}} \right\}$$

$$\Rightarrow Df = \left[ -\frac{1}{\sqrt[3]{1}} \right) \cup \left[ \frac{1}{\sqrt[3]{1}}, +\infty \right) \cup (-\infty, \frac{1}{\sqrt[3]{1}}] \cup \left[ \frac{1}{\sqrt[3]{1}}, +\infty \right)$$

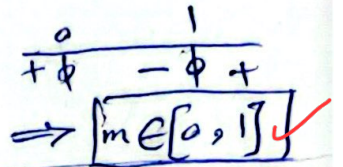


$$f(x) = \sqrt{mx^2 + 5mx + 1}$$

(5)  $\checkmark$

$$Df = \mathbb{R} \begin{cases} m > 0 \\ \Delta \leq 0 \end{cases}$$

$$\Delta \leq 0 \Rightarrow 25m^2 - 4m \leq 0 \Rightarrow m(m-1) \leq 0$$



$$f(x) = \begin{cases} \frac{x^2 - 1}{x^2 + 1} & ; x \neq \pm 1 \\ x + k & ; x = \pm 1 \end{cases}$$

$$g(x) = x + 1$$

$$f(x) = g(x) \Rightarrow Dg = Df$$

$$Dg = \mathbb{R} \Rightarrow Df = \mathbb{R} \Rightarrow a = \frac{1}{\sqrt[3]{1}} \Rightarrow \frac{1}{\sqrt[3]{1}}$$

$$g\left(\frac{1}{\sqrt[3]{1}}\right) = f\left(\frac{1}{\sqrt[3]{1}}\right) \Rightarrow \frac{1}{\sqrt[3]{1}} + 1 = \frac{1}{\sqrt[3]{1}} + k \Rightarrow k = 0$$

$$\Rightarrow k = 0 \quad \boxed{a + k = \frac{1}{\sqrt[3]{1}}} \quad a + k = \frac{1}{\sqrt[3]{1}}$$

$$f(x) = \begin{cases} \frac{9x^2 - 5}{x^2 + 5} & ; x \neq -\frac{5}{9} \\ 5a + 5 & ; x = -\frac{5}{9} \end{cases}$$

$$f(x) = g(x) \quad f(1) = g(1) = \frac{9 - 5}{1 + 5} = 1 = 5a + 5$$

$$\rightarrow b = -5$$

$$f\left(-\frac{5}{9}\right) = g\left(-\frac{5}{9}\right) = -5 - 5 = -10 = 5a + 5 \Rightarrow a = -2$$

$$g(x) = 5x + b \quad \boxed{a - b = 5 - (-5) = 10}$$

$$f(x) = \begin{cases} \frac{x^2 - 5}{x - 5} & ; x \neq 5 \\ 5a^2 + 9x & ; x = 5 \end{cases}$$

$$f(x) = g(x) = f(5) = g(5) = 5 + 5 = 10$$

$$f = 5a^2 + 9a$$

$$\Rightarrow 5a^2 + 9a - 10 = 0 \rightarrow a^2 + 5a - 10 = 0$$

$$g(x) = x + 5$$

$$\rightarrow a^2 + 5a - 10 = (a + 5)(a - 2) = 0 \Rightarrow \boxed{a = -2}$$

$$\boxed{a = 1}$$