

$$\frac{a=a}{\implies} a^r + r a = a^{-r} \implies a = -r \quad (1)$$

$$f(r) = r \implies \frac{f+a}{f-b} = r \implies 1 \implies f+a \implies a=1 \quad f(1) = \frac{1+1}{1+1} = \frac{1^r}{1} = 1$$

$$g(r) = r \implies f+b = r \implies b = -1$$

$$f(x) = \frac{rx+1}{rx^2+ax+b} \quad (x+1)(x-r) = x^2 - rx - r \xrightarrow{x^2} rx^2 - rx - r \implies a = -r, b = -1 \quad (2)$$

$$f(1) = \frac{1}{1-1-1} = \frac{1}{-1}$$

$$f(x) = \frac{x^r - \sqrt{x}}{-rx^2+ax+b} \quad (x+1)^r = x^r + rx + 1 \xrightarrow{x-r} -rx^r - rx - r \implies a = -r, b = -r \quad (3)$$

$$f(x) = \frac{rx}{(x-1)(x^2+mx+1)} \quad \Delta < 0 \implies m^2 - 4 < 0 \implies m^2 < 4 \implies -2 < m < 2 \quad m \in (-2, 2)$$

$$x=1: r+m=0 \implies m = -r$$

$$f(x) = \sqrt{r - \frac{1}{x^r}} \quad x \neq 0 \quad r - \frac{1}{x^r} > 0 \implies r > \frac{1}{x^r} \implies r > \frac{1}{x} > -r \implies x > \frac{1}{r}, -\frac{1}{r} > x$$

$$x \in (-\infty, -\frac{1}{r}] \cup [\frac{1}{r}, \infty) \quad (4)$$

$$f(x) = \sqrt{mx^r + rmx+1} \quad mx^r + rmx+1 > 0 \quad a > 0 \implies m > 0 \quad m \in [0, 1]$$

$$\Delta < 0 \implies r^2 m^2 - 4m < 0 \implies m(m-4) < 0 \quad \frac{0}{+} \quad \frac{1}{-} \quad \frac{1}{+}$$

$$a = \frac{1}{r} \quad g(x) = f(x) \xrightarrow{x=\frac{1}{r}} r = r+k \implies k=0 \quad a+k = \frac{1}{r} \quad (5)$$

$$\frac{rx^r - r}{rx^2 + r} = rx + b \implies b = -r \quad rx^2 + r = rx - r \xrightarrow{x=\frac{1}{r}} -ra = -r \implies a = r$$

$$a-b = r - (-r) = 2r$$

$$x+r = ra^r + a \xrightarrow{x=r} r = r(a^r + a) \implies r = a(a+1) \implies a=1, a=-r \quad (6)$$