

$$a^2 + 2a = a^2 - 4 \Rightarrow 2a = -4$$

$$\Rightarrow a = \boxed{-2}$$

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$$f(x) = \frac{x^2 + a}{2x - b} \Rightarrow f(x) = \frac{x^2 + a}{2x + 1} \xrightarrow{(2, 3)} 3 = \frac{4 + a}{5} \Rightarrow 4 + a = 15 \Rightarrow a = 11$$

$$g(x) = 2x + b \xrightarrow{(2, 3)} 3 = 2(2) + b \Rightarrow b = -1$$

$$f(x) = \frac{x^2 + 11}{2x + 1} \longrightarrow f(1) = \frac{12}{3} = \boxed{4}$$

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$$f(x) = \frac{4x + 1}{2x^2 + ax + b}$$

$$D_f = \mathbb{R} - \{-1, 1\} \rightarrow \text{ریشه های مخرج} \Rightarrow 2(x+1)(x-1) = 2x^2 - 2x - 2$$

$$f(x) = \frac{4x + 1}{2x^2 - 2x - 2} \longrightarrow f(1) = \frac{5}{-12}$$

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$$f(x) = \frac{2x^2 - 5x}{-2x^2 + ax + b}$$

$$D_f = \mathbb{R} - \{-1\} \rightarrow \text{ریشه مخرج} \Rightarrow -2(x+1)^2 = -2x^2 - 4x - 2 \Rightarrow a = -4, b = -2$$

$$a + b = -4 - 2 = \boxed{-6}$$

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$$f(x) = \frac{2x}{(x-1)(x^2 + mx + 1)}$$

$$D_f = \mathbb{R} - \{1\} \rightarrow \text{ریشه مخرج} \Rightarrow \begin{cases} x^2 + mx + 1 = (x-1)^2 \Rightarrow m = -2 \\ \Delta < 0 \Rightarrow m^2 - 4 < 0 \Rightarrow (m+2)(m-2) < 0 \Rightarrow -2 < m < 2 \end{cases}$$

$\frac{-2 \quad 2}{+ \quad - \quad - \quad +}$

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$$f(x) = \sqrt{x - \frac{1}{x^2}}$$

$$1) x \neq 0 \quad 2) x - \frac{1}{x^2} \geq 0 \Rightarrow \frac{x^3 - 1}{x^2} \geq 0 \Rightarrow \frac{(x-1)(x+1)}{x^2} \geq 0$$

$$\frac{-\frac{1}{x} \cdot \frac{1}{x}}{-\phi + \phi - \phi +} \Rightarrow D_f = \left[-\frac{1}{x} ; 0\right) \cup \left[\frac{1}{x} ; +\infty\right)$$

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$$f(x) = \sqrt{mx^2 + 1}$$

$$D_f = \mathbb{R} \begin{cases} m > 0 \\ \Delta \leq 0 \end{cases} \Rightarrow m^2 - 1 \leq 0 \Rightarrow m(m-1) \leq 0 \quad \frac{0}{+\phi - \phi +} \Rightarrow [0 ; 1]$$

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$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} ; x \neq 1 \\ x + k ; x = 1 \end{cases}$$

$$f(x) = g(x) \Rightarrow \begin{cases} Dg = Df \\ Dg = \mathbb{R} \end{cases} \left. \begin{array}{l} \\ \end{array} \right\} D_f = \mathbb{R} \Rightarrow a = \frac{1}{x} \rightarrow \text{ریشه منفرجه}$$

$$g(x) = x + 1$$

$$g\left(\frac{1}{x}\right) = f\left(\frac{1}{x}\right) \Rightarrow x\left(\frac{1}{x}\right) + 1 = \left(\frac{1}{x}\right) + k \Rightarrow x = \frac{1}{x} + k \Rightarrow k = 0$$

$$a + k = \frac{1}{x}$$

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$$f(x) = \begin{cases} \frac{9x^2 - 4}{x + 2} ; x \neq -\frac{2}{3} \\ 2ax + 2 ; x = -\frac{2}{3} \end{cases}$$

$$f(x) = g(x)$$

$$f\left(-\frac{2}{3}\right) = g\left(-\frac{2}{3}\right) = \frac{9 - 4}{-\frac{2}{3} + 2} = 1 = 2a + b \Rightarrow b = -2$$

$$f\left(-\frac{2}{3}\right) = g\left(-\frac{2}{3}\right) = -2 - 2 = -4 = -2a + 2 \Rightarrow a = 3$$

$$g(x) = 2ax + b$$

$$a - b = 3 - (-2) = 5$$

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$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} ; x \neq 2 \\ \lambda a^x + a ; x = 2 \end{cases}$$

$$f(x) = g(x) \Rightarrow f(2) = g(2) = 2 + 2 = 4 = \lambda a^2 + a$$

$$\Rightarrow \lambda a^2 + a - 4 = 0 \longrightarrow a^2 + \lambda a - 4 = (a + \lambda)(a - 2) = 0 \Rightarrow \begin{cases} a = \frac{-\lambda}{1} = -\lambda \\ a = \frac{2}{1} = 2 \end{cases}$$

$$g(x) = x + \lambda$$

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