

الف) $y = \sqrt{\frac{x^2 - 4x}{x^3 - x}} = \sqrt{\frac{x(x-4)}{x(x-1)(x+1)}} \Rightarrow Df = (-1, 1) \cup (4, +\infty) - \{0\}$

ب) $y = \sqrt{\frac{x^5 - x^3}{x^3 - x}} = \sqrt{\frac{x^3(x+1)(x-1)}{x(x+1)(x-1)}} \Rightarrow Df = \mathbb{R} - \{-1, 1\}$

الف) $y = \frac{x+1}{\sqrt{x+|x|}} \Rightarrow x+|x| > 0 \Rightarrow x > 0 \Rightarrow Df = (0, +\infty)$

ب) $y = \frac{1}{\sqrt{x|x|}} \Rightarrow x > 0$ if $x=0$ تعریف نشده $x > 0$ تعریف شده $\Rightarrow Df = (0, +\infty)$

الف) $y = \sqrt{2-[x]} \Rightarrow 2-[x] \geq 0 \Rightarrow 2 \geq [x] \Rightarrow 3 > x \Rightarrow Df = (-\infty, 3)$

ب) $y = \frac{1}{\sqrt{2-[x]}} \Rightarrow 2-[x] > 0 \Rightarrow 2 > [x] \Rightarrow 2 > x \Rightarrow Df = (-\infty, 2)$

الف) $y = \sqrt{[x-\frac{1}{2}] + [x+\frac{1}{2}]} \Rightarrow [x+\frac{1}{2}] = [x-\frac{1}{2}] + 1$
 $2[x-\frac{1}{2}] + 1 \geq 0 \Rightarrow [x-\frac{1}{2}] \geq -\frac{1}{2} \Rightarrow [x-\frac{1}{2}] \geq 0 \Rightarrow x-\frac{1}{2} \geq 0 \Rightarrow x \geq \frac{1}{2} \Rightarrow Df = [\frac{1}{2}, +\infty)$

ب) $y = \sqrt{[x-\frac{1}{2}] + [x+\frac{1}{2}]} = \sqrt{[a] + [-a]} \Rightarrow [a] + [-a] \geq 0 \Rightarrow \begin{cases} a \in \mathbb{Z} \Rightarrow [a] + [-a] = 0 \\ a \notin \mathbb{Z} \Rightarrow [a] + [-a] < 0 \end{cases}$
 $\rightarrow x - \frac{1}{2} = k \Rightarrow x = k + \frac{1}{2} \Rightarrow Df = \{x | x = k + \frac{1}{2}, k \in \mathbb{Z}\}$

الف) $y = \frac{2}{\sin x - 1} \Rightarrow \sin x \neq 1 \Rightarrow x \neq 2k\pi + \frac{\pi}{2}$
 $Df = \mathbb{R} - \{2k\pi + \frac{\pi}{2} | k \in \mathbb{W}\}$

ب) $y = \frac{2}{2\sin^2 x - 1} \Rightarrow 2\sin^2 x \neq 1 \Rightarrow \sin^2 x \neq \frac{1}{2} \Rightarrow \sin x \neq \pm \frac{1}{\sqrt{2}}$
 $\Rightarrow x \neq \frac{k\pi}{2} + \frac{\pi}{4} \Rightarrow Df = \mathbb{R} - \{\frac{k\pi}{2} + \frac{\pi}{4} | k \in \mathbb{W}\}$

$$\text{أ) } y = \sqrt{2 \sin x - 1} \Rightarrow 2 \sin x \geq 1 \Rightarrow \sin x \geq \frac{1}{2} \Rightarrow x = \left[2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6} \right] \\ \Rightarrow Df = \left[2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6} \right]$$

$$\text{ب) } \sqrt{\cos x - 1} \Rightarrow \cos x \geq 1 \Rightarrow x = \{2k\pi\} \\ Df = \{2k\pi\}$$

$$\text{أ) } \cos y = |x| - 3 \quad -1 \leq |x| - 3 \leq 1 \Rightarrow 2 \leq |x| \leq 4 \Rightarrow \begin{cases} 2 \leq x \leq 4 \\ -4 \leq x \leq -2 \end{cases} \\ Df = [-4, -2] \cup [2, 4]$$

$$\text{ب) } y = \arcsin(x^2 + 3x + 1) \quad -1 \leq x^2 + 3x + 1 \leq 1 \Rightarrow \begin{cases} x^2 + 3x + 1 \leq 1 \Rightarrow x^2 + 3x \leq 0 \\ x^2 + 3x + 1 \geq -1 \Rightarrow x^2 + 3x + 2 \geq 0 \end{cases} \\ Df = [-2, -1] \cup [0, 1]$$

$$\text{أ) } y = \sqrt{2 - \log_2(x-2)} \quad x-2 > 0 \Rightarrow x > 2 \Rightarrow 2 - \log_2(x-2) \geq 0 \Rightarrow 2 \geq \log_2(x-2) \Rightarrow x-2 \leq 4 \Rightarrow x \leq 6 \\ 2 > x-2 \Rightarrow 4 > x \\ Df = (2, 6]$$

$$\text{ب) } y = \log_2(x^3 - x) \quad x^3 - x > 0 \quad x(x^2 - 1) > 0 \Rightarrow \begin{cases} x > 1 \\ 0 < x < 1 \end{cases} \\ Df = (-1, 0) \cup (1, +\infty)$$

$$\text{أ) } y = \frac{2^x}{2^x - 1} \quad 2^x \neq 1 \\ Df = \mathbb{R}$$

$$\text{ب) } y = \frac{2^x}{2^x - 2} \quad 2^x - 2 \neq 0 \quad 2^x \neq 2 \Rightarrow Df = \mathbb{R} - \{1, 2\}$$

$$\text{أ) } y = (fx + 1)! \quad k \in \mathbb{W}$$

$$fx + 1 = k \Rightarrow$$

$$fx = k - 1 \Rightarrow$$

$$x = \frac{k-1}{f}$$

$$Df = \left\{ x \mid x = \frac{k-1}{f}, k \in \mathbb{W} \right\}$$

$$\text{ب) } y = \frac{2^x}{2^x - 1} \quad 2^x - 1 \neq 0 \quad 2^x \neq 1 \quad x \neq 0 \\ Df = \mathbb{R} - \{0\}$$

$$\text{ج) } y = \frac{2^x}{2^x - 2} \quad 2^x - 2 \neq 0 \quad 2^x \neq 2 \Rightarrow Df = \mathbb{R} - \{1, 2\}$$

$$\text{ب) } y = \left(\frac{2^x - 2}{2^x - 1} \right)! \quad k \in \mathbb{W}$$

$$\frac{2^x - 2}{2^x - 1} = k \Rightarrow$$

$$2^x - 2 = k(2^x - 1) \Rightarrow$$

$$2^x - k2^x = -k + 2 \Rightarrow$$

$$(1-k)2^x = 2 - k$$

$$x = \frac{\log(2-k)}{\log(1-k)}$$

$$Df = \left\{ x \mid x = \frac{\log(2-k)}{\log(1-k)}, k \in \mathbb{W} \right\}$$