

الف) $y = \sqrt{\frac{x^2 - 4x}{x^2 - 4}} = \sqrt{\frac{x(x-4)}{(x+2)(x-2)}}$ $\begin{matrix} -1 & 0 & 4 \\ + & - & + & - & + \end{matrix} \Rightarrow D_f = (-3, 0) \cup (0, 4) \cup (4, +\infty)$

ب) $y = \sqrt{\frac{x^3 - 2x^2}{x^2 - x}} = \sqrt{\frac{x^2(x-2)(x-1)}{x(x+1)(x-1)}}$ $\begin{matrix} -1 & 0 & 1 \\ + & - & + & - & + \end{matrix} \Rightarrow D_f = \mathbb{R} - \{-1, 0, 1\}$

الف) $y = \frac{x+1}{\sqrt{x+1|x|}} \rightarrow \begin{matrix} x+1 > 0 \\ |x| > -x \end{matrix} \Rightarrow D_f = \mathbb{R}^+ \setminus (0, +\infty)$

ب) $y = \frac{1}{\sqrt{|x|x|}} \rightarrow \begin{matrix} |x|x| > 0 \\ \oplus \vee \ominus \times \ominus \times \oplus \end{matrix} \Rightarrow D_f = \mathbb{R}^+ \setminus (0, +\infty)$

الف) $y = \sqrt{2-x} \rightarrow 2-[x] \geq 0 \quad [x] \leq 2 \Rightarrow D_f = (-\infty, 3)$

ب) $y = \frac{1}{\sqrt{2-[x]}} \rightarrow 2-[x] > 0 \quad [x] < 2 \Rightarrow D_f = (-\infty, 2)$

الف) $y = \sqrt{[x - \frac{1}{3}] + [x + \frac{2}{3}]} = \sqrt{[\frac{2x+1}{3}] + [\frac{x+2}{3}]} = \sqrt{[x + \frac{2}{3}] + 1 + [x + \frac{2}{3}]}$
 $\rightarrow 2[x + \frac{2}{3}] + 1 \geq 0 \Rightarrow [x + \frac{2}{3}] \geq -\frac{1}{2} \Rightarrow x + \frac{2}{3} \geq -\frac{1}{2} \Rightarrow x \geq -\frac{7}{6} \Rightarrow D_f = [-\frac{7}{6}, +\infty)$

ب) $y = \sqrt{[x - \frac{1}{3}] + [-x + \frac{1}{3}]} \rightarrow \begin{cases} a \in \mathbb{Z} & [a] + [-a] = 0 \\ a \notin \mathbb{Z} & [a] + [-a] = -1 \end{cases} \Rightarrow x - \frac{1}{3} \in \mathbb{Z} \Rightarrow D_f = \{x | x = k + \frac{1}{3}, k \in \mathbb{Z}\}$

الف) $y = \frac{2}{\sin x - 1} \rightarrow \sin x \neq 1 \Rightarrow D_f = \mathbb{R} - \{2k\pi + \frac{\pi}{2}\}$

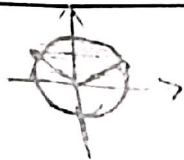
ب) $y = \frac{3}{2\sin x - 1} \rightarrow 2\sin x - 1 \neq 0 \Rightarrow \sin x \neq \pm \frac{1}{2} \Rightarrow \sin x \neq \pm \frac{\sqrt{p}}{p}$
 $\Rightarrow D_f = \mathbb{R} - \{\frac{k\pi}{2} + \frac{\pi}{6}, \frac{k\pi}{2} + \frac{5\pi}{6}\}$

$$\text{ب) } y = \sqrt{r \sin x - 1} \rightarrow r \sin x - 1 \geq 0 \rightarrow \sin x \geq \frac{1}{r}$$

$$\Rightarrow D_f = \left[2k\pi + \frac{\pi}{2}, 2k\pi + \frac{3\pi}{2} \right]$$

$$\text{ج) } y = \sqrt{\cos x - 1} \rightarrow \cos x - 1 \geq 0 \Rightarrow \cos x \geq 1$$

$$\Rightarrow D_f = \{2k\pi\}$$



$$\text{د) } \cos y = |x| - r$$

$$\begin{aligned} |x| - r \leq 1 & \quad |x| \leq r+1 \rightarrow -r-1 \leq x \leq r+1 \\ |x| - r \geq -1 & \quad |x| \geq r-1 \rightarrow x \geq r-1 \text{ or } x \leq -r+1 \end{aligned}$$

$$\Rightarrow D_f = [-r-1, r+1] \cup [r-1, \infty) \cup (-\infty, -r+1]$$

$$\text{هـ) } y = \arcsin(x^2 + 3x + 1)$$

$$-1 \leq x^2 + 3x + 1 \leq 1 \rightarrow x^2 + 3x \leq 0 \rightarrow x(x+3) \leq 0$$

$$\left\{ \begin{aligned} x^2 + 3x + 1 \geq -1 & \rightarrow x^2 + 3x + 2 \geq 0 \rightarrow (x+1)(x+2) \geq 0 \\ x^2 + 3x + 1 \leq 1 & \rightarrow x^2 + 3x \leq 0 \rightarrow x(x+3) \leq 0 \end{aligned} \right.$$

$$y = \sqrt{r - \log_p(x^2)} \quad x-2 > 0 \rightarrow x > 2$$

$$r - \log_p(x-2) \geq 0 \rightarrow \log_p(x-2) \leq r \rightarrow x-2 \leq p^r \rightarrow x \leq 2 + p^r$$

$$\Rightarrow D_f = [2, 2 + p^r]$$

$$y = \log(x^2 - x) \rightarrow x^2 - x > 0$$

$$\frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow D_f = \left(\frac{-1 + \sqrt{5}}{2}, \infty \right) \cup \left(\frac{-1 - \sqrt{5}}{2}, 0 \right)$$

$$\text{و) } x^n + 1 \neq 0 \rightarrow x^n \neq -1 \checkmark \Rightarrow D_f = \mathbb{R}$$

$$\text{ب) } x^n - 1 \neq 0 \rightarrow x^n \neq 1 \Rightarrow x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$$

$$\text{ج) } x^n - y \neq 0 \rightarrow x^n \neq y \Rightarrow D_f = \mathbb{R} - \{\log y\}$$

$$\text{د) } x^n - y^2 \neq 0 \rightarrow x^n \neq y^2 \Rightarrow D_f = \mathbb{R} - \{\log y^2\}$$

$$\text{هـ) } y = (f(x+1))! \rightarrow f(x+1) \in \mathbb{N} \rightarrow f(x) \in \mathbb{N} - 1$$

$$x \in \frac{\mathbb{N}-1}{f} \Rightarrow D_f = \left\{ x \mid x = \frac{k-1}{f}, k \in \mathbb{N} \right\}$$

$$\rightarrow y = \left(\frac{r(x-y)}{x-y} \right)! \rightarrow \frac{r(x-y)}{x-y} \in \mathbb{N} \rightarrow x \in \frac{-a\mathbb{N} + r}{r - y}$$

$$\Rightarrow D_f = \left\{ x \mid x = \frac{-ak + r}{r - y}, k \in \mathbb{N} \right\}$$