

$$y = \sqrt{x f(x)} \rightarrow \begin{cases} x=0 \\ f(x)=0 \rightarrow x=0 \text{ و } 2 \text{ و } 4 \end{cases}$$

$x \mid -\infty \quad 0 \quad 2 \quad 4$

$\rightarrow D_f = [-\infty, 0] \cup [2, 4]$

$$y = \sqrt{\frac{-x}{f(x)}} \rightarrow \begin{cases} x=0 \\ f(x)=0 \rightarrow x=0 \text{ و } 2 \text{ و } -3 \text{ و } 0 \end{cases}$$

$x \mid -\infty \quad 0 \quad 2 \quad -3 \quad 0$

$\rightarrow D_f = (-\infty, 0) \cup (0, 2) \cup (2, -3) \cup (-3, 0)$

سبب، عدد صحیح

$$f(x) - 2f(2) = x - 2x + 4$$

$$x=2 \rightarrow f(2) - 2f(2) = 4 - 4 + 4 = 4$$

$$-f(2) = 4 \rightarrow f(2) = -4$$

$$x=-2 \rightarrow f(-2) - 2f(-2) = 4 - 4 + 4 = 4 \rightarrow f(-2) = 4$$

$$f(f(x)) \rightarrow f(1) = -2x + 3 = 1$$

$$f(x) = 1 - 2x + 3 = 2 \Rightarrow f(f(1)) = 2$$

$$f(f(x)) \rightarrow f(2) = 2x + 3 = 2 \Rightarrow f(f(x)) = 2 \Rightarrow f(2) + f(x) = 2x + 3 = 2 \Rightarrow f(x) = 2 - 2x - 3 = -2x - 1$$

$$f(x) = ax^2 - bx + c$$

$$f(x-1) = a(x-1)^2 - b(x-1) + c = ax^2 - 2ax + a - bx + b + c = ax^2 - (2a+b)x + (a+b+c)$$

$$f(x-1) - f(x) = ax^2 - (2a+b)x + (a+b+c) - (ax^2 - bx + c) = -2ax + a - bx + b = -2ax - bx + a + b = -(2a+b)x + (a+b)$$

$$-(2a+b)x + (a+b) = 0 \Rightarrow -(2a+b)x = -(a+b) \Rightarrow (2a+b)x = a+b \Rightarrow x = \frac{a+b}{2a+b}$$

$$a-b = 2 \Rightarrow a-b = -1$$

$$f(u) = u^2 - 2u + 1$$

$$f(3) = 9 - 6 + 1 = 4$$

$$f(-1) = 1 + 2 + 1 = 4$$

$$f(2) = 4 - 4 + 1 = 1$$

$$\frac{f(3) + f(-1)}{f(2)} = \frac{4 + 4}{1} = 8 \quad \boxed{8}$$

$$f(x) \rightarrow x^2 - 2x + 1$$

$$f(-x) \rightarrow (-x)^2 - 2(-x) + 1 = x^2 + 2x + 1$$

$$f(x) \rightarrow x^2 - 2x + 1$$

$$\frac{f(x) - f(-x)}{f(x)} = \frac{x^2 - 2x + 1 - (x^2 + 2x + 1)}{x^2 - 2x + 1} = \frac{-4x}{x^2 - 2x + 1} \quad \boxed{\frac{-4x}{x^2 - 2x + 1}}$$

$$\frac{f(f(x)) - f(f(y))}{f(x) - y} = \frac{9 - 4}{1} = 5 \quad \boxed{5}$$

$$f(u) = u^2 - 2u + 1$$

$$f(\sqrt{u} - 1) = \frac{u - 2\sqrt{u} + 1}{u + 1} = \frac{(\sqrt{u} - 1)^2}{u + 1}$$

$$f(f(\sqrt{u} - 1)) = \frac{1}{u + 1} \quad \boxed{\frac{1}{u + 1}}$$

$$f(u) = u^2 + \frac{1}{u^2}$$

$$f(\sqrt{u}) = (\sqrt{u})^2 + \frac{1}{(\sqrt{u})^2} = u + \frac{1}{u}$$

$$(f(\sqrt{u}))^2 = \left(u + \frac{1}{u}\right)^2 = u^2 + \frac{1}{u^2} + 2 = f(u) + 2 \quad \boxed{f(u) + 2}$$