نام و نام خانوادگی امیرعباس عارج ۱۹/۵ پاسخنامه تشریحی تکلیف شماره ..۲۵.. کلاس .... B - دهم ریاضی

1

الف) $(\sin 135^\circ + \sin 300^\circ)(\cos 315^\circ - \cos 150^\circ) = \left(\frac{\sqrt{2}-\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}+\sqrt{3}}{2}\right) = \frac{2-3}{4} = -\frac{1}{4}$

$\sin 135^\circ = \frac{\sqrt{2}}{2}$, $\sin 300^\circ = -\frac{\sqrt{3}}{2}$ → $\left(\frac{\sqrt{2}-\sqrt{3}}{2}\right)$; $\cos 315^\circ = +\frac{\sqrt{2}}{2}$, $-\cos 150^\circ = -\left(-\frac{\sqrt{3}}{2}\right)$ → $\left(\frac{\sqrt{2}+\sqrt{3}}{2}\right)$

ب) $\frac{1+3\tan^2 210^\circ}{4\cos^2 330^\circ + 2\cos^2 225^\circ} = \frac{1+3\left(\frac{1}{\sqrt{3}}\right)^2}{4\left(\frac{\sqrt{3}}{2}\right)^2+2\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{1+1}{3+1} = \frac{2}{4} = \frac{1}{2}$

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2

الف) $AH = BA \times \sin 45^\circ = \sqrt{6} \times \frac{\sqrt{2}}{2} = \sqrt{3} \Rightarrow AH = \sqrt{3}$

$AC^2 = AH^2 + HC^2 \Rightarrow HC^2 = (2\sqrt{3})^2 - (\sqrt{3})^2 = 12 - 3 = 9 \Rightarrow HC = \sqrt{9} = 3$

ب) $\frac{HC}{AC} = \cos 30^\circ \Rightarrow AC = \frac{9}{\frac{\sqrt{3}}{2}} = \frac{18}{\sqrt{3}} = 6\sqrt{3} \Rightarrow AH = 6\sqrt{3} \times \sin 30^\circ = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$ (وتر AC)

$\sin(B) = \frac{AH}{AB} = \frac{3\sqrt{3}}{3\sqrt{6}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \hat{B} = 45^\circ$

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3

الف) $50\sqrt{3} = \text{وتر} \times \sin 60^\circ \Rightarrow AD = \frac{50\sqrt{3}}{\frac{\sqrt{3}}{2}} = 100 \Rightarrow \text{وتر} \times \cos 60^\circ = BD \Rightarrow BD = \frac{100}{2} = 50$

$AC \times \sin 30^\circ = 50\sqrt{3} \Rightarrow AC = \frac{50\sqrt{3}}{\frac{1}{2}} = 100\sqrt{3} \Rightarrow 100\sqrt{3} \times \cos 30^\circ = BC \Rightarrow$ (وتر)

$BC = 150 = (50 + x) \Rightarrow x = 100$

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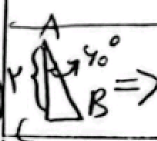
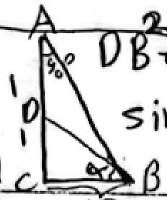
ب)  $\Rightarrow AB \times \cos 60^\circ = 2 \Rightarrow AB = \frac{2}{\frac{1}{2}} = 4 \Rightarrow$ (وتر)

$BC = \text{وتر} \times \sin 60^\circ = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$



$DB^2 = (2\sqrt{3})^2 + (1)^2 = 13 \Rightarrow DB = \sqrt{13}$

$\sin\alpha = \frac{CD}{DB} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

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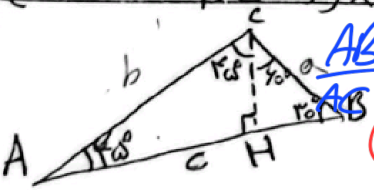

$\frac{AB}{AC} = \frac{a(\sqrt{6}+\sqrt{2})}{4a} = \frac{\sqrt{6}+\sqrt{2}}{4}$

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$\frac{AB}{AC} = \sqrt{2-\sqrt{3}}$

$\frac{CB}{\sin(45^\circ)} = \frac{AC}{\sin(105^\circ)} = \frac{AB}{\sin 30^\circ} \Rightarrow 2AB = \frac{AC}{\sin(105^\circ)} \Rightarrow$

$\left.\begin{array}{l} AB = c \\ CB = a \\ AC = b \end{array}\right\} \Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow$

$AB = \frac{AC}{2\sin(105^\circ)} \Rightarrow \frac{AB}{AC} = \frac{\frac{AC}{2\sin(105^\circ)}}{AC} = \frac{1}{2\sin(105^\circ)}$

$\sin(105^\circ) = \pm\sqrt{\frac{1-\cos 210^\circ}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}}$ (ربع دوم)

$\frac{AB}{AC} = \frac{1}{2\sin(105^\circ)} = \frac{1}{2 \times \frac{\sqrt{2+\sqrt{3}}}{2}} = \frac{1}{\sqrt{2+\sqrt{3}}} \times \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}} = \frac{\sqrt{2-\sqrt{3}}}{1} = \sqrt{2-\sqrt{3}}$

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$\frac{S_{\hat{ABE}}}{S_{\hat{BCD}}} = \frac{\frac{1}{2} \times AB \times BE \times \sin(B)}{\frac{1}{2} \times DB \times CB \times \sin(B)} = \frac{2 \times 4 \times \sin(B)}{3 \times 6 \times \sin(B)} = \frac{4}{9}$

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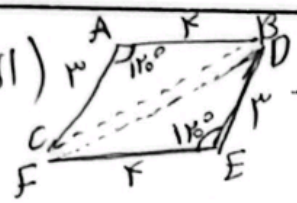
الف) 

$\Rightarrow S_{ABCD} = 2 \times S_{ABC} \Rightarrow S_{ABC} = \frac{1}{2} \times 4 \times 3 \times \sin(120^\circ) = \frac{12\sqrt{3}}{4} \Rightarrow$

$\sin(120^\circ) = \frac{\sqrt{3}}{2}$

$S_{ABCD} = 2 \times \frac{12\sqrt{3}}{4} = 6\sqrt{3}$ ✓

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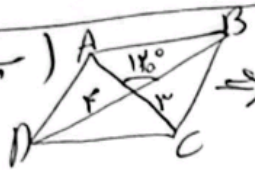
ب) 

$\Rightarrow S_{ABCD} = \frac{1}{2} \times 3 \times 4 \times \sin(120^\circ) = 3\sqrt{3}$

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$Q = (3, 4)$, $P = (-1, 1)$ $\Rightarrow m = \frac{\Delta y}{\Delta x} = \frac{4-1}{3-(-1)} = \frac{3}{4} = \frac{1}{2} \Rightarrow$

$\tan\theta = \frac{1}{2} \Rightarrow 1 + \tan^2(\theta) = \frac{1}{\cos^2\theta} \Rightarrow 1 + \frac{1}{4} = \frac{5}{4} = \frac{1}{\cos^2\theta} \Rightarrow$

$\cos^2\theta = \frac{4}{5} \Rightarrow \cos\theta = \sqrt{\frac{4}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ ✓

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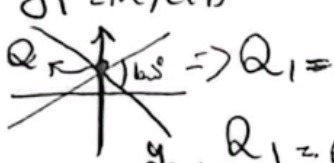
$2y_1 = 2x + 5 \Rightarrow y_1 = x + \frac{5}{2} \rightarrow y_1$ شیب خط $= 1 \Rightarrow \tan\alpha = 1 \Rightarrow \alpha = 45^\circ \Rightarrow$

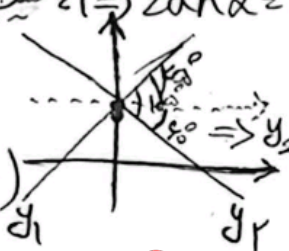
$y_2 = mx + b$



$\Rightarrow Q_1 = (0, \frac{5}{2}), Q_2 = (0, b)$

$Q_1 = Q_2 \Rightarrow b = \frac{5}{2}$

$y_2 = mx + \frac{5}{2}$



y_2 شیب خط $= \tan(-60^\circ) = -\sqrt{3} \Rightarrow$

$m = -\sqrt{3} \Rightarrow$

$y_2 = -\sqrt{3}x + \frac{5}{2} \Rightarrow$

۲ $m \times b = -\sqrt{3} \times \frac{5}{2} = \frac{-5\sqrt{3}}{2}$ ✓

$F(x) = \cos(\pi + x) + \sin(\frac{\pi}{2} - x) - \tan(\frac{3\pi}{2} + x) \Rightarrow$

$F(x) = -\cos(x) + \cos(x) - (-\cot(x)) = \cot(x) \Rightarrow$

$F(x) = \cot(x) \Rightarrow F(\frac{5\pi}{4}) = \cot(\frac{5\pi}{4}) = \cot(135^\circ) = -1$ ✓

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$\sin(\frac{17\pi}{2} + x) = \sin(\frac{16\pi}{2} + \frac{\pi}{2} + x) = \sin(\frac{\pi}{2} + x) = \cos(x)$

$\cos(\frac{15\pi}{2} - x) = \cos(\frac{14\pi}{2} + \frac{\pi}{2} - x) = -\cos(\frac{\pi}{2} - x) = -\sin(x)$

$\cos(3\pi + x) = -\cos(x)$ / $\sin(4\pi - x) = \sin(x)$

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$\Rightarrow \frac{\sin(\frac{17\pi}{2} + x) + \cos(\frac{15\pi}{2} - x)}{\cos(3\pi + x) - \sin(4\pi - x)} = \frac{\cos x - \sin x}{-\cos x - (-\sin x)} = \frac{\cos x - \sin x}{-(\cos x - \sin x)}$

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$= -1$ ✓