

$$1) (\sin 135^\circ + \sin 225^\circ) (\cos 135^\circ - \cos 225^\circ) = \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right) \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) = \frac{2-2}{2} = 0$$

$$2) \frac{1 + \tan^2 210^\circ}{4 \cos^2 330^\circ + 2 \cos^2 225^\circ} = \frac{1 + \left(\frac{\sqrt{3}}{2} \right)^2}{4 \left(\frac{\sqrt{3}}{2} \right)^2 + 2 \left(\frac{\sqrt{2}}{2} \right)^2} = \frac{1 + \frac{3}{4}}{3 + 1} = \frac{\frac{7}{4}}{4} = \frac{7}{16}$$

$$AH = BA \times \sin 45^\circ = \sqrt{4} \times \frac{\sqrt{2}}{2} = \sqrt{2} \Rightarrow AH = \sqrt{2}$$

$$AC^2 = AH^2 + HC^2 \Rightarrow HC^2 = (2\sqrt{2})^2 - (\sqrt{2})^2 = 12 - 2 = 10 \Rightarrow HC = \sqrt{10}$$

$$\frac{HC}{AC} = \cos 30^\circ \Rightarrow AC = \frac{10}{\frac{\sqrt{3}}{2}} = \frac{20}{\sqrt{3}} \Rightarrow AH = \frac{20}{\sqrt{3}} \times \sin 30^\circ = \frac{10}{\sqrt{3}}$$

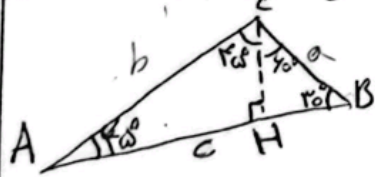
$$\sin(B) = \frac{AH}{AB} = \frac{\frac{10}{\sqrt{3}}}{2\sqrt{3}} = \frac{1}{2} \Rightarrow B = 30^\circ$$

$$50\sqrt{2} \times \sin 45^\circ \Rightarrow AD = \frac{50\sqrt{2}}{\frac{\sqrt{2}}{2}} = 100 \Rightarrow 100 \times \cos 45^\circ = BD \Rightarrow BD = \frac{100}{\sqrt{2}} = 50\sqrt{2}$$

$$AE \times \sin 30^\circ = 50\sqrt{2} \Rightarrow AC = \frac{50\sqrt{2}}{\frac{1}{2}} = 100\sqrt{2} \Rightarrow \frac{100\sqrt{2}}{\sqrt{2}} \times \cos 30^\circ = BC \Rightarrow$$

$$BC = 100 \times (\cos 30^\circ) = 50\sqrt{3}$$

$$AB \times \cos 30^\circ = 100 \Rightarrow AB = \frac{100}{\frac{\sqrt{3}}{2}} = \frac{200}{\sqrt{3}} \Rightarrow \frac{AB}{AC} = \frac{\frac{200}{\sqrt{3}}}{100\sqrt{2}} = \frac{2}{\sqrt{6}} = \frac{\sqrt{6}}{3}$$



$$\frac{CB}{\sin(45^\circ)} = \frac{AC}{\sin(105^\circ)} = \frac{AB}{\sin(30^\circ)} \Rightarrow 2AB = \frac{AC}{\sin(105^\circ)}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{a}{\sin 105^\circ} = \frac{b}{\sin 30^\circ} = \frac{c}{\sin 45^\circ}$$

$$\sin(105^\circ) = \sin(45^\circ + 60^\circ) = \sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ = \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2}(1+\sqrt{3})}{4}$$

$$\frac{AB}{AC} = \frac{\sin(105^\circ)}{\sin(30^\circ)} = \frac{\frac{\sqrt{2}(1+\sqrt{3})}{4}}{\frac{1}{2}} = \frac{\sqrt{2}(1+\sqrt{3})}{2}$$

$$\frac{S_{ABE}}{S_{BCD}} = \frac{AB \times BE \times \sin(B)}{DB \times CB \times \sin(B)} = \frac{AB \times BE}{DB \times CB} = \frac{AB}{DB} \times \frac{BE}{CB} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$\therefore 1) \Rightarrow r \times S_{ABC} \Rightarrow S_{ABC} = \frac{1}{2} \times r \times r \times \sin(120^\circ) = \frac{1}{2} \times \frac{r^2}{\sqrt{3}} \Rightarrow$
 $S_{ABC} = \frac{r^2}{2\sqrt{3}}$

$\therefore 2) \Rightarrow S_{ABCD} = \frac{1}{2} \times r \times r \times \sin(120^\circ) = \boxed{\frac{r^2}{2\sqrt{3}}}$

$Q = (1, \frac{1}{r})$
 $P = (-1, 1) \Rightarrow m = \frac{\Delta y}{\Delta x} = \frac{1 - \frac{1}{r}}{1 - (-1)} = \frac{1}{r} = \frac{1}{r} \Rightarrow$

$\tan \theta = \frac{1}{2} \Rightarrow 1 + \tan^2(\theta) = \frac{1}{\cos^2 \theta} \Rightarrow 1 + \frac{1}{r^2} = \frac{1}{\cos^2 \theta} \Rightarrow$
 $\cos^2 \theta = \frac{r^2}{r^2 + 1} \Rightarrow \cos \theta = \frac{r}{\sqrt{r^2 + 1}} = \boxed{\frac{r}{\sqrt{r^2 + 1}}}$

$r y_1 = r x_1 + a \Rightarrow y_1 = x_1 + \frac{a}{r} \rightarrow y_1 = x_1 + \frac{a}{r} \Rightarrow \tan \alpha = 1 \Rightarrow \alpha = 45^\circ \Rightarrow$
 $y_1 = m x_1 + b$
 $Q_1 = (0, \frac{a}{r})$ and $Q_2 = (0, b)$
 $Q_1 = Q_2 \Rightarrow b = \frac{a}{r}$
 $y_2 = m x_2 + b$
 $y_2 = m x_2 + \frac{a}{r}$
 $m = -\sqrt{r} \Rightarrow y_2 = -\sqrt{r} x_2 + \frac{a}{r} \Rightarrow$
 $m \times b = -\sqrt{r} \times \frac{a}{r} = \boxed{-\frac{a\sqrt{r}}{r}}$

$F(x) = \cos(x + \frac{\pi}{4}) + \sin(\frac{\pi}{4} - x) = \tan(\frac{\pi}{4} + x) \Rightarrow$
 $F(x) = -\cos(x) + \cos(x) - (-\cot(x)) = \cot(x) \Rightarrow$
 $F(x) = \cot(x) \Rightarrow F(\frac{\omega \pi}{4}) = \cot(\frac{\omega \pi}{4}) = \cot(180^\circ) = \boxed{-\sqrt{r}}$

$\sin(\frac{11\pi}{4} + x) = \sin(\frac{14\pi}{4} + \frac{\pi}{4} + x) = \sin(\frac{\pi}{4} + x) = \cos(x)$
 $\cos(\frac{15\pi}{4} - x) = \cos(\frac{14\pi}{4} + \frac{\pi}{4} - x) = \cos(\frac{\pi}{4} - x) = -\sin(x)$
 $\cos(3\pi + x) = -\cos(x) / \sin(4\pi - x) = \sin(x)$

$\Rightarrow \frac{\sin(\frac{11\pi}{4} + x) + \cos(\frac{15\pi}{4} - x)}{\cos(3\pi + x) - \sin(4\pi - x)} = \frac{\cos x - \sin x}{-\cos x - (-\sin x)} = \frac{\cos x - \sin x}{-\cos x + \sin x} = \boxed{-1}$