نیما محقق دهم ریاضی B سری ۲۰

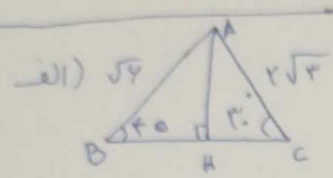
۱ـ

الف) $(\sin ۱۳۵ + \sin ۳۰۰)(\cos ۳۱۵ - \cos ۱۵۰) = \left(\frac{\sqrt{۲}}{۲} + \left(-\frac{\sqrt{۳}}{۲}\right)\right)\left(\frac{\sqrt{۲}}{۲} - \left(-\frac{\sqrt{۳}}{۲}\right)\right)$

$= \left(\frac{\sqrt{۲}}{۲}\right)^۲ - \left(\frac{\sqrt{۳}}{۲}\right)^۲ = \frac{۲}{۴} - \frac{۳}{۴} = -\frac{۱}{۴}$

ب) $\dfrac{۱ + ۳\tan^۲ ۲۱۰}{۴\cos^۲ ۳۳۰ + ۲\cos^۲ ۲۲۵} = \dfrac{۱ + ۳\times\left(\frac{\sqrt{۳}}{۳}\right)^۲}{۴\times\left(\frac{\sqrt{۳}}{۲}\right)^۲ + ۲\times\left(\frac{\sqrt{۲}}{۲}\right)^۲} = \dfrac{۱ + ۳\times\frac{۱}{۳}}{۴\times\frac{۳}{۴} + ۲\times\frac{۲}{۴}} = \dfrac{۱+۱}{۳+۱} = \frac{۲}{۴} = \frac{۱}{۲}$

۲ـ

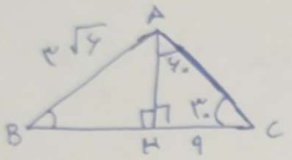


الف) $AH = BH$

$AH = \sqrt{۶}\times \sin ۴۵ = \sqrt{۶}\times\frac{\sqrt{۲}}{۲} = \frac{\sqrt{۱۲}}{۲}$

$AH = ۲\sqrt{۳}\times\sin\hat{C} \Rightarrow \frac{\sqrt{۱۲}}{۲} \div \sqrt{۱۲} = \sin\hat{C} \Rightarrow \sin\hat{C} = \frac{۱}{۲} \Rightarrow \hat{C} = ۳۰$

$\Rightarrow HC = ۲\sqrt{۳}\times\frac{\sqrt{۳}}{۲} = ۳$

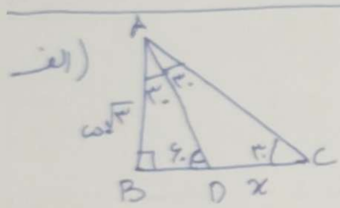


ب) $AH = ۹\times\sin ۳۰ \Rightarrow AH = \frac{۹}{\sqrt{۳}}$

$AH = ۳\sqrt{۶}\times\sin B \Rightarrow \frac{۹}{\sqrt{۳}} = ۳\sqrt{۶}\times\sin\hat{B}$

$\Rightarrow \sin\hat{B} = \frac{\sqrt{۲۷}}{\sqrt{۵۴}} \Rightarrow \sin\hat{B} = \frac{۱}{\sqrt{۲}} = \frac{\sqrt{۲}}{۲} \Rightarrow \hat{B} = ۴۵$

۳ـ

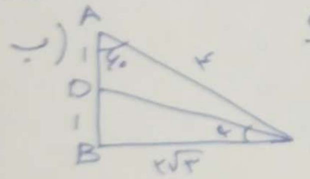


الف) $AB = AD\times\frac{\sqrt{۳}}{۲} \Rightarrow ۵۰\sqrt{۳} = AD\times\frac{\sqrt{۳}}{۲} \Rightarrow AD = 100$
($\sin ۶۰$)

$AB = AC\times\sin ۳۰ \Rightarrow AC = ۵۰\sqrt{۳}\div\frac{۱}{۲} \Rightarrow AC = ۱۰۰\sqrt{۳}$

$CB - BD = x \Rightarrow ۱۵۰ - ۵۰ = ۱۰۰$

$CB = ۱۰۰\sqrt{۳}\times\frac{\sqrt{۳}}{۲} = ۱۵۰ \quad , \quad BD = ۱۰۰\times\frac{۱}{۲} = ۵۰$

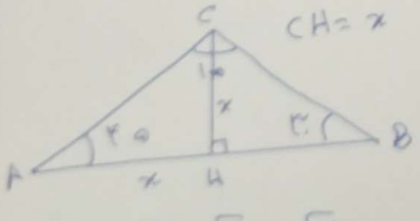


ب) $\hat{C} = ۳۰ , AB = ۲ \Rightarrow AB = AC\times\sin\hat{C} \Rightarrow AC = ۲\div\frac{۱}{۲} = ۴$

$CB = AC\times\cos\hat{C} \Rightarrow CB = ۴\times\frac{\sqrt{۳}}{۲} = ۲\sqrt{۳}$

$\sin\alpha = \frac{۱}{DC} \Rightarrow \sin\alpha = \frac{۱}{\sqrt{۱+۱۲}} = \frac{۱}{\sqrt{۱۳}} = \frac{\sqrt{۱۳}}{۱۳}$

۴ـ



$CH = x \Rightarrow AC\times\frac{\sqrt{۲}}{۲} = x \Rightarrow AC = \sqrt{۲}x$

$AB = AH + HB \Rightarrow AB = x + \sqrt{۳}x \Rightarrow AB = x(۱+\sqrt{۳})$

$\frac{AB}{AC} = \frac{\sqrt{۲}x}{(۱+\sqrt{۳})x}$

$HB = ۲x\times\frac{\sqrt{۳}}{۲} = \sqrt{۳}x$

$CB = ۲x$

-۵

$S_{BCD} = 3\times 6\times \frac{1}{2}\times \sin\hat{B}$

$S_{ABE} = 2\times 4\times \frac{1}{2}\times \sin\hat{B}$

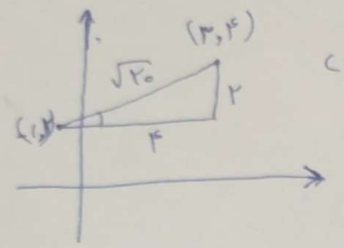
$\frac{S_{BCD}}{S_{ABE}} = \frac{3\times 6\times \frac{1}{2}\times \sin\hat{B}}{2\times 4\times \frac{1}{2}\times \sin\hat{B}} = \frac{9}{4} = 2/25$

-۶

الف) $2\times 3\times 4\times \sin 120^\circ \times \frac{1}{2} = 3\times 4\times \frac{\sqrt{3}}{2} = 6\sqrt{3}$

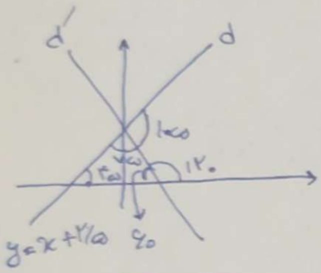
ب) $3\times 4\times \sin 120^\circ = \frac{\sqrt{3}}{2}\times 12 = 6\sqrt{3}$

-۷



$\cos x = \frac{4}{\sqrt{20}} = \frac{2}{\sqrt{5}}$

-۸



$d = x + 2/5$

$a = 1 \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45$

$d' = mx + b \Rightarrow d' = -\sqrt{3}x + b$

$\tan 120$

$2/5 \longrightarrow$ تقاطع برخورد با خط d در $x = 0$

$m \times b = -2/5\sqrt{3}$

-۹

$f(x) = \cos(\pi + x) + \sin(\frac{\pi}{2} - x) - \tan(\frac{3\pi}{2} - x) \Rightarrow f(x) = \underbrace{-\cos x + \cos x}_{0} + \cot x$

$\cos(\pi + x) \to -\cos x$, $\sin(\frac{\pi}{2} - x) \to \cos x$, $\tan(\frac{3\pi}{2} - x) \to \cot x$

$\Rightarrow f(\frac{5\pi}{6}) = \cot(\frac{5\pi}{6}) = -\sqrt{3}$

-۱۰

$\frac{\sin(\frac{17\pi}{2} + x) + \cos(\frac{15\pi}{2} - x)}{\cos(3\pi + x) - \sin(6\pi - x)} = \frac{\sin x + \cos x}{\sin x - \cos x}$

$\cos(3\pi + x) \to -\cos x$, $\sin(6\pi - x) \to \sin x$

$\sin(\frac{17\pi}{2} + x) = \cos x$

$\cos(\frac{15\pi}{2} - x) = \sin x$