نام و نام خانوادگی ...آرشین جهانی... ۴ پاسخنامه تشریحی تکلیف شماره ..۲۱.. کلاس ...دهم ریاضی B...

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$(1!+3!\times5!, \dots +25!)$ فرد

$1+6+0,0,0$ یکان ۷

$(2!+4!\times6!\times\dots\times241)$ زوج $= 6\times7=42$

$\frac{1\times2}{2}+4+0,\dots$

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$(4+\sqrt7)^{-1}(1+\sqrt7)^2 = \frac{1}{4+\sqrt7}\times 1+2\sqrt7+7 = \frac{16+4+8\sqrt7+28+\sqrt7+14\sqrt7}{4+\sqrt7}$ ۰/۵

$\sqrt[4]{(4+\sqrt7)^{-1}}\times\sqrt{1+\sqrt7} = \sqrt[4]{\frac{(1-\sqrt7)^{-1}}{18}}\times\sqrt{1+\sqrt7} = \frac{\sqrt{\sqrt7-1}\sqrt{\sqrt7+1}}{\sqrt[4]{18}} = \frac{\sqrt6}{\sqrt[4]{18}} = \frac{\sqrt[4]{36}}{\sqrt[4]{18}} = \sqrt[4]{2}$ $= \frac{4\sqrt7+4\sqrt7}{4+\sqrt7}$

ب) $\underbrace{\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)}_{a}\underbrace{\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)}_{b} \to a=\frac{(\sqrt2+\sqrt5)}{\sqrt2(\sqrt2+\sqrt5)}=\frac{1}{\sqrt2}$

$b^2=((3-\sqrt5)+(3+\sqrt5)-2\sqrt{9-5})=6-4=2 \to b^2=2 \to b=\begin{cases}+\sqrt2 & \times\\ -\sqrt2 & \checkmark\end{cases}$ $ab=\frac{1}{\sqrt2}\times(-\sqrt2)=-1$

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الف) $\frac{\sqrt8+\sqrt{27}}{5-\sqrt6}-2(\sqrt[4]{9}-1)^{-1}$ $\frac{\sqrt8+\sqrt{27}}{5-\sqrt6}=\frac{2\sqrt2+3\sqrt3}{5-\sqrt6}\times\frac{5+\sqrt6}{5+\sqrt6}=\frac{10\sqrt2+4\sqrt3+15\sqrt3+9\sqrt2}{19}=\frac{19\sqrt2+19\sqrt3}{19}=\sqrt2+\sqrt3$

$(\sqrt[4]{9}-1)^{-1}=\frac{1}{\sqrt3-1}\times\frac{\sqrt3+1}{\sqrt3+1}=\frac12(\sqrt3+1) \to (\sqrt2+\sqrt3)-2\left(\frac12(\sqrt3+1)\right)=\sqrt2-1$

ب) $\frac{\sqrt{27}-1}{4+\sqrt3}=\frac{3\sqrt3-1}{4+\sqrt3}\times\frac{4-\sqrt3}{4-\sqrt3}=\frac{12\sqrt3-9-4+\sqrt3}{13}=\frac{13\sqrt3-13}{13}=\sqrt3-1$ $(2-\sqrt3)^{-1}=\frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3}=2+\sqrt3$

$\to \sqrt3-1+2+\sqrt3=2\sqrt3+1$

$\frac{3\sqrt3-1}{4+\sqrt3}+\frac{1}{2-\sqrt3}=\frac{6\sqrt3-2-9+2\sqrt3}{8+2\sqrt3-4\sqrt3-3}=\frac{7\sqrt3-11}{5-2\sqrt3}$ ب)

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الف) $\frac{\overbrace{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}^{A}}{\sqrt{\sqrt3-\sqrt2}}-2 \longrightarrow \sqrt6+2-2=\sqrt6$

$A^2=(\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1})^2=1+\sqrt3+\sqrt3-1+2\sqrt{3-1}=2\sqrt3+2\sqrt2 \longrightarrow A=\sqrt2\left(\sqrt{\sqrt3+\sqrt2}\right)$

$\frac{\sqrt2\left(\sqrt{\sqrt3+\sqrt2}\right)}{\left(\sqrt{\sqrt3-\sqrt2}\right)}=\sqrt2\times\sqrt{\frac{\sqrt3+\sqrt2}{\sqrt3-\sqrt2}} \longrightarrow \frac{\sqrt3+\sqrt2}{\sqrt3-\sqrt2}\times\frac{\sqrt3+\sqrt2}{\sqrt3+\sqrt2}=(\sqrt3+\sqrt2)^2 \longrightarrow \sqrt2\times\sqrt{(\sqrt3+\sqrt2)^2}=\sqrt2(\sqrt3+\sqrt2)$

ب) $\sqrt[12]{2^8}\times3\sqrt[4]{2}\times2\sqrt[12]{2}=\sqrt[3]{4}\times3\sqrt[4]{2}\times2\sqrt[12]{2}=2^{\frac23}\times3\times2^{\frac14}\times2^{\frac1{12}}=3\times2^{1+\frac23+\frac14+\frac1{12}}=3\times2^{\frac{24}{12}}=3\times2^2=12$

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$\frac{3^a(1+3+3^2+\dots+3^5)}{2^{a-2}(1+2+2^2+\dots+2^5)} \to \frac{3^a(364)}{2^a\times2^{-2}\times63}=52 \to \left(\frac32\right)^a\times\frac{4\times364}{63}=52 \to \left(\frac32\right)^a=\frac94 \to a=2$

$1+3+3^2+\dots+3^5 \longrightarrow q=3,\ a=1$ دنباله هندسی $\longrightarrow S_n=a_1\frac{(1-q^n)}{1-q} \longrightarrow S_n=1\times\frac{(1-3^6)}{1-3}=364$

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$1+2+2^2+\dots+2^5 \longrightarrow q=2,\ a=1$ دنباله هندسی $\longrightarrow S_n=1\times\frac{(1-2^6)}{(1-2)}=63$

$\left(\sqrt[4]{\sqrt{6}-2}-\sqrt[4]{\sqrt{6}+2}\right)^{2}\left(\sqrt{\sqrt{6}-2}+\sqrt{\sqrt{6}+2}\right)^{2}=t\,(2-\sqrt{3})$ ۰.۲۵

$\left(\left(\sqrt[4]{\sqrt{6}-2}\right)^{2}-\left(\sqrt[4]{\sqrt{6}+2}\right)^{2}\right)^{2}=\sqrt{6}-2+\sqrt{6}+2-2(6-4)$ ۶

$2\sqrt{6}-4$

$4(8-4\sqrt{3})=16(2-\sqrt{3})\rightarrow t=16$

$\sqrt[4]{\sqrt{6}-2}\times\sqrt[4]{\sqrt{6}+2}=(6-4)$

$\left(\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2}\right)^{2}=\left(\sqrt{6}-2+\sqrt{6}+2-2\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)}\right)^{2}=\left(2\sqrt{6}-2\sqrt{2}\right)^{2}=4(6+2-2\sqrt{12})$

$(\underbrace{a+\frac{1}{a}}_{a}+\underbrace{\sqrt{2}}_{b})^{2}(\underbrace{a+\frac{1}{a}}_{a}-\underbrace{\sqrt{2}}_{b})^{2}=2^{t}$

$\left((a+\frac{1}{a})^{2}-2\right)^{2}=\left(a^{2}+\frac{1}{a^{2}}+2-2\right)^{2}=\left(a^{2}+\frac{1}{a^{2}}\right)^{2}=a^{4}+\frac{1}{a^{4}}+2$

$\left(a^{2}+\frac{1}{a^{2}}+2-2+2\right)^{2}=a^{4}+\frac{1}{a^{4}}+2=2^{t}$ ۰ ۷

$7-4\sqrt{3}+\frac{1}{7-4\sqrt{3}}+2=9-4\sqrt{3}+\left(\frac{1}{7-4\sqrt{3}}\times\frac{7+4\sqrt{3}}{7+4\sqrt{3}}\right)=$

$7-4\sqrt{3}+\frac{1}{7-4\sqrt{3}}+2=2^{t}\Rightarrow 49-48+1+2=2^{t}$

$4=2^{t}\Rightarrow t=2$

$9-4\sqrt{3}+\frac{7+4\sqrt{3}}{49-48}=16=2^{4}=2^{t}\rightarrow t=4$

$\left(\frac{1}{2}\right)^{-\frac{4}{3}}=\sqrt[3]{24}=2\sqrt[3]{3}$ $\quad\sqrt[3]{16}=2\sqrt[3]{2}$

۰ $\sqrt[5]{32}\times2\sqrt[3]{2}\times2\sqrt[3]{2}=2^{\frac{2}{3}}\times2^{\frac{1}{3}}\times2^{\frac{2}{5}}=\;2^{\frac{10+10+6}{15}}=2^{\frac{26}{15}}$ ۸

$\sqrt[15]{2^{26}}=2\sqrt[15]{2^{11}}$

$A=(\underbrace{2^{2}\times2^{\frac{4}{3}}}_{2^{\frac{10}{3}}})^{\frac{1}{5}}\times2^{\frac{4}{3}}=2^{\frac{2}{3}}\times2^{\frac{4}{3}}=4$

$\frac{26}{15}\quad\frac{5}{15}\quad(2\times\frac{1}{2})$

$(2\sqrt[15]{2^{11}})^{\frac{1}{3}}\quad\frac{11}{15}$

$\boxed{\left(2^{\frac{1}{15}}\sqrt{2}\right)^{-\frac{1}{3}}}$

$\rightarrow(2A)^{-\frac{1}{3}}=8^{-\frac{1}{3}}=\frac{1}{\sqrt[3]{8}}=\frac{1}{2}$

$\sqrt[7]{a}=27\times a^{\frac{15}{7}}\rightarrow\sqrt[7]{a}=27\times\sqrt[7]{a^{15}}$

۰ $\sqrt[7]{a}=27\times\sqrt[7]{a^{15}}$

$\rightarrow a^{2}=\frac{1}{27}\rightarrow a=\frac{1}{\sqrt{27}}=\frac{1}{3\sqrt{3}}$

$a^{\frac{1}{7}}=3^{3}\times a^{\frac{15}{7}}$ ۹

$a^{\frac{1}{7}-\frac{14}{7}}$

$\rightarrow\left(\frac{1}{a}-3\right)=3\sqrt{3}-3=3(\sqrt{3}-1)\rightarrow\frac{3(\sqrt{3}-1)}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}=\frac{3(1+3-2\sqrt{3})}{3-1}=\frac{3(4-2\sqrt{3})}{2}=$

$3(2-\sqrt{3})=6-3\sqrt{3}$

$(\sqrt{x+a}+\sqrt{x-4})(\underbrace{\sqrt{x+a}-\sqrt{x-4}}_{2})=(x+a)-(x-4)=a+4$

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$\rightarrow\frac{a}{2}+2\;,\;\frac{a+4}{2}$

$\sqrt{x+a}+\sqrt{x-4}=\frac{a}{2}+2\rightarrow\sqrt{x+a}+\sqrt{x-4}-2=\frac{a}{2}+2-2=\frac{a}{2}$