

نام و نام خانوادگی: پارسا... ۱۹/۵ پاسخنامه تشریحی تکلیف شماره ۱! کلاس دهم، پارسا...

$$(1 + \sqrt{2} + \sqrt{3} + \dots + 0) (\sqrt{2} + \sqrt{3} + \dots + 0) = \dots \quad \checkmark$$

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الف) $(\sqrt{2})^2 = 2\sqrt{2} = 2(\sqrt{2})$
 $\Rightarrow \sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2}$
 $= \sqrt{4}$ ✓

ب) $\sqrt{2} = \frac{\sqrt{2}+1}{\sqrt{2}} \Rightarrow \frac{\sqrt{2}+1}{\sqrt{2}} - \frac{\sqrt{2}+1}{\sqrt{2}}$
 $= -\sqrt{2} \Rightarrow \frac{\sqrt{2}+\sqrt{2}}{\sqrt{2}+1} \times (-\sqrt{2}) = -1$ ✓

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الف) $\frac{\sqrt{2}+\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2\sqrt{2}+2}{1} = 2\sqrt{2}+2$
 $\frac{2}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \sqrt{2}+1$ ۱۵
 $\Rightarrow \sqrt{2} + \sqrt{2} - \sqrt{2} - 1 = \sqrt{2} - 1$ ✓

ب) $\sqrt[3]{2} = 2^{\frac{1}{3}}$
 $\sqrt[3]{12} = 2^{\frac{1}{3}} \times 3^{\frac{1}{3}}$
 $\sqrt[3]{4} = 2^{\frac{2}{3}}$
 $\Rightarrow 2^{\frac{1}{3}} \times 2^{\frac{2}{3}} \times 3^{\frac{1}{3}} = 2^{\frac{1}{3}+\frac{2}{3}} \times 3^{\frac{1}{3}} = 2 \times 3^{\frac{1}{3}} = 2\sqrt[3]{3}$ ✓

الف) $(\sqrt{2} + \sqrt{3} - 1)^2 = 1 + \sqrt{2} + \sqrt{3} - 1$ ۲
 $+ 2\sqrt{2}(\sqrt{3}-1) = 2\sqrt{2} + 2\sqrt{3} \Rightarrow \sqrt{2} + \sqrt{3}$
 $= \sqrt{2} \sqrt{2} + \sqrt{3} \sqrt{3} \Rightarrow \sqrt{2} \sqrt{3} = \frac{1}{\sqrt{2} + \sqrt{3}} \Rightarrow \sqrt{2} \sqrt{3} + \sqrt{2} \sqrt{3}$
 $\Rightarrow \sqrt{2} \sqrt{3} = \sqrt{2} (\sqrt{3} + \sqrt{3}) = \sqrt{2} + 2 \Rightarrow \sqrt{2} + 2 = \sqrt{2}$ ✓

ب) $\frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{2}+1}$
 $\frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}+1}{1}$ ۱
 $\frac{\sqrt{2}-1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2-1}{1} = 1$
 $\Rightarrow 1 - \sqrt{2} + 2 + \sqrt{2} = 3$ ۲
 $(\sqrt{2} + \sqrt{3}) - 2(\frac{1}{\sqrt{2}+1}) = \sqrt{2} - 1$ ۳

$\frac{2^m (1 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)}{2^m (2^{-1} + 2^{-2} + 1 + 2^1 + 2^2 + 2^3)} = \frac{2^m \times 24}{2^m \times \frac{92}{2}} \Rightarrow \frac{2^m}{2^m} \times 24 \times \frac{2}{92} = 52$
 $\Rightarrow \frac{2^m}{2^m} \times \frac{24}{92} \times \frac{2}{1} = 1 \Rightarrow \frac{2^m}{2^m} \times \frac{2}{9} = 1 \Rightarrow \frac{2^m}{2^m} = \frac{9}{2} \Rightarrow 2 = 3$ ۲ ✓

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$$((a-b)^r)^r ((a+b)^r)^r = (a-b)^r (a+b)^r \Rightarrow (a^r - b^r)^r = a^r - b^r = \sqrt{4-r} - \sqrt{4+r}$$

$$\Rightarrow \sqrt{4-r} - \sqrt{4+r} = -\sqrt{4} \quad (\sqrt{4-r})(\sqrt{4+r}) = r \Rightarrow (\sqrt{4-r} - \sqrt{4+r})^r = r^r + 1 - 1 = r^r$$

$$\Rightarrow 2r - 19\sqrt{r} = 19(5-\sqrt{r}) \Rightarrow r = 19$$

Y

$$((a + \frac{1}{a})^r - r)^r \Rightarrow a^r + \frac{1}{a^r} + r - r \Rightarrow (a^r + \frac{1}{a^r})^r \Rightarrow \sqrt{r-4}\sqrt{r} = \sqrt{(r-4)r} \Rightarrow a^r = r\sqrt{r}$$

$$\Rightarrow a^r + \frac{1}{a^r} = r - \sqrt{r} + r + \sqrt{r} = r \Rightarrow r = 19 \quad 19 = r^r \Rightarrow r = 19$$

Y

$$r^{\frac{1}{n}} \times r^{\frac{1}{n}} = r^{\frac{2}{n}} \quad r^{\frac{1}{n}} \times \frac{1}{r^{\frac{1}{n}}} = r^{\frac{1}{n} - \frac{1}{n}} = r^0 = 1$$

$$(\frac{1}{r})^{-\frac{1}{n}} = (r^{-1})^{-\frac{1}{n}} = r^{\frac{1}{n}} \Rightarrow r^{\frac{1}{n}} \times r^{\frac{1}{n}} = r^{\frac{2}{n}} = r$$

$$(rA)^{-\frac{1}{n}} = (r \times A)^{-\frac{1}{n}} = r^{-\frac{1}{n}} = \frac{1}{r^{\frac{1}{n}}} = \frac{1}{r}$$

Y

$$a^{\frac{1}{n}} = r \vee a^{\frac{1}{n}} \Rightarrow a = r^n \times a^{\frac{n}{n}} \Rightarrow a = r^n a \Rightarrow r^n a - a = 0 \quad a^r = \frac{1}{r} \rightarrow a = \frac{1}{r^n} = \frac{1}{r^n}$$

$$a(r^n - 1) = 0 \Rightarrow a = \frac{0}{r^n - 1} = 0$$

$$\frac{\sqrt{r}-r}{\sqrt{r}+1} \times \frac{\sqrt{r}-1}{\sqrt{r}-1} = \frac{-r\sqrt{r}+r}{r} = -\sqrt{r}+1$$

$$\rightarrow (\frac{1}{a} - r) = r\sqrt{r} - r = r(\sqrt{r}-1) \rightarrow \frac{r(\sqrt{r}-1)}{\sqrt{r}+1} \times \frac{\sqrt{r}-1}{\sqrt{r}-1} = \frac{r(1-r-\sqrt{r})}{r-1} = \frac{r(1-r\sqrt{r})}{r} = r(1-\sqrt{r}) = 4-19\sqrt{r}$$

$$\sqrt{x+a} + \sqrt{x-r} = r \quad \sqrt{x+a} - r = \sqrt{x-r} \Rightarrow \sqrt{x+a} > \sqrt{x-r} + r$$

$$\sqrt{x-r} + r + \sqrt{x-r} - r = r\sqrt{x-r}$$

$$(\sqrt{x+a} + \sqrt{x-r})(\sqrt{x+a} - \sqrt{x-r}) = (x+a) - (x-r) = a+r$$

$$\rightarrow \frac{a}{r} + r \mid a+r$$

$$\sqrt{x+a} + \sqrt{x-r} = \frac{a}{r} + r \rightarrow \sqrt{x+a} + \sqrt{x-r} - r = \frac{a}{r} + r - r = \frac{a}{r}$$