

نام و نام خانوادگی پاسخنامه تشریحی تکلیف شماره ۱۰۰۰ کلاس دهم پسران چابک...

$$(1 + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5} + \dots + \sqrt{100})^2 = \dots = 2$$

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الف) $(\sqrt{2})^2 = 2\sqrt{2} = 2(\sqrt{2})$
 $\Rightarrow \sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2}$
 $= \sqrt{4}$

ب) $\sqrt{2} = \frac{\sqrt{2}+1}{\sqrt{2}} \Rightarrow \frac{\sqrt{2}+1}{\sqrt{2}} - \frac{\sqrt{2}+1}{\sqrt{2}}$
 $= -\sqrt{2} \Rightarrow \frac{\sqrt{2}+\sqrt{2}}{\sqrt{2}+1} \times (-\sqrt{2}) = -1$

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الف) $\frac{\sqrt{2}+\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2\sqrt{2}+2}{1} = 2\sqrt{2}+2$
 $\frac{2}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \sqrt{2}+1$
 $\Rightarrow \sqrt{2} + \sqrt{2} - 1 = \sqrt{2} - 1$

ب) $\sqrt[4]{2} = 2^{\frac{1}{4}}$
 $\sqrt[4]{16} = 2 \times 2^{\frac{1}{4}}$
 $\sqrt[4]{4} = 2^{\frac{1}{2}}$
 $\Rightarrow 2^{\frac{1}{4}} \times 2^{\frac{1}{4}} \times 2^{\frac{1}{4}} \times 2^{\frac{1}{4}} = 2 \times 2 = 4 \times 2 = 8$

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الف) $(\sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8} + \sqrt{9} + \sqrt{10})^2 = 1 + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8} + \sqrt{9} + \sqrt{10}$
 $+ 2(\sqrt{2}\sqrt{3} + \sqrt{2}\sqrt{4} + \sqrt{2}\sqrt{5} + \sqrt{2}\sqrt{6} + \sqrt{2}\sqrt{7} + \sqrt{2}\sqrt{8} + \sqrt{2}\sqrt{9} + \sqrt{2}\sqrt{10} + \sqrt{3}\sqrt{4} + \sqrt{3}\sqrt{5} + \sqrt{3}\sqrt{6} + \sqrt{3}\sqrt{7} + \sqrt{3}\sqrt{8} + \sqrt{3}\sqrt{9} + \sqrt{3}\sqrt{10} + \sqrt{4}\sqrt{5} + \sqrt{4}\sqrt{6} + \sqrt{4}\sqrt{7} + \sqrt{4}\sqrt{8} + \sqrt{4}\sqrt{9} + \sqrt{4}\sqrt{10} + \sqrt{5}\sqrt{6} + \sqrt{5}\sqrt{7} + \sqrt{5}\sqrt{8} + \sqrt{5}\sqrt{9} + \sqrt{5}\sqrt{10} + \sqrt{6}\sqrt{7} + \sqrt{6}\sqrt{8} + \sqrt{6}\sqrt{9} + \sqrt{6}\sqrt{10} + \sqrt{7}\sqrt{8} + \sqrt{7}\sqrt{9} + \sqrt{7}\sqrt{10} + \sqrt{8}\sqrt{9} + \sqrt{8}\sqrt{10} + \sqrt{9}\sqrt{10})$
 $= \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8} + \sqrt{9} + \sqrt{10}$
 $\Rightarrow \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8} + \sqrt{9} + \sqrt{10} = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8} + \sqrt{9} + \sqrt{10}$

ب) $\frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}+1}{\sqrt{2}-1} + \frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{2-1}{\sqrt{2}-1} + \frac{2-1}{\sqrt{2}+1} = \frac{2}{\sqrt{2}-1} + \frac{2}{\sqrt{2}+1} = \frac{2(\sqrt{2}+1) + 2(\sqrt{2}-1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{2\sqrt{2}+2+2\sqrt{2}-2}{2-1} = \frac{4\sqrt{2}}{1} = 4\sqrt{2}$
 $\Rightarrow 1 - \sqrt{2} + 2 + \sqrt{2} = 3$

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$\frac{2^m(1+2^1+2^2+2^3+2^4+2^5)}{2^m(2^{-1}+2^{-2}+2^{-3}+2^{-4}+2^{-5})} = \frac{2^m \times 246}{2^m \times \frac{96}{5}} \Rightarrow \frac{2^m}{2^m} \times 246 \times \frac{5}{96} = 12.5$
 $\Rightarrow \frac{2^m}{2^m} \times 246 \times \frac{5}{96} \times \frac{1}{1} = 1 \Rightarrow \frac{2^m}{2^m} \times \frac{1}{9} = 1 \Rightarrow \frac{2^m}{2^m} = \frac{9}{1} \Rightarrow 2^m = 9$

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$((a-b)^{\sqrt{2}})^{\sqrt{2}} ((a+b)^{\sqrt{2}})^{\sqrt{2}} = (a-b)^2 (a+b)^2 \Rightarrow (a^2 - b^2)^{\sqrt{2}} = a^{\sqrt{2}} - b^{\sqrt{2}} = \sqrt{4-5} - \sqrt{4+5}$ $\Rightarrow \cancel{(\sqrt{4-5})^{\sqrt{2}} (\sqrt{4+5})^{\sqrt{2}}} \sqrt{4-5} + \sqrt{4+5} = 2\sqrt{4} \quad (\sqrt{4-5}) (\sqrt{4+5}) = 2 \Rightarrow (\sqrt{4-5} \sqrt{4+5})^{\sqrt{2}} = 2^{\sqrt{2}+1-1} = 2^{\sqrt{2}}$ $\Rightarrow 2^{\sqrt{2}} - 19\sqrt{4} = 19(5-\sqrt{4}) \Rightarrow t = 19$	6
$(1 + \frac{1}{a})^{\sqrt{2}-2} \Rightarrow a^{\sqrt{2}} \frac{1}{a^2} \sqrt{2}-2 \Rightarrow (a^{\sqrt{2}} \frac{1}{a^2})^{\sqrt{2}} \Rightarrow \sqrt{2-4\sqrt{2}} = \sqrt{2-4\sqrt{2}} \Rightarrow a^{\sqrt{2}} = \sqrt{2}\sqrt{2}$ $\Rightarrow a^{\sqrt{2}} \frac{1}{a^2} = 2 - \sqrt{2} \sqrt{2} + \sqrt{2} = 2 \Rightarrow 2^{\sqrt{2}} = 19 \quad 19 = 2^{\sqrt{2}} \Rightarrow t = 2$	7
$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^{\frac{1}{2}} \quad \text{and} \quad x^{\frac{1}{2}} \times \frac{1}{x^{\frac{1}{2}}} = x^{\frac{1}{2}-\frac{1}{2}} = x^0 = \sqrt[4]{x^2 \sqrt{4}}$ $(\frac{1}{2})^{-\frac{1}{2}} = (2^{-1})^{-\frac{1}{2}} = 2^{\frac{1}{2}} \Rightarrow x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^{\frac{1}{2}+\frac{1}{2}} = x$ $(2A)^{-\frac{1}{2}} = (2 \times 2)^{-\frac{1}{2}} = 2^{-\frac{1}{2}} = \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$	8
$a^{\frac{1}{\sqrt{2}}} = \sqrt{2} \times a^{\frac{10}{\sqrt{2}}} \Rightarrow a = \sqrt{2}^{\frac{\sqrt{2}}{\sqrt{2}}} \times a^{\frac{\sqrt{2}}{\sqrt{2}}} \Rightarrow a = 2a^{\sqrt{2}} \Rightarrow 2a^{\sqrt{2}} - a = 0$ $a(2a^{\sqrt{2}} - 1) = 0 \Rightarrow a < \frac{0}{\frac{1}{\sqrt{2}}} \times \frac{\sqrt{2}-2}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{-2\sqrt{2}+4}{1} = -2\sqrt{2}+4$	9
$\sqrt{x+a} + \sqrt{x-b} = 7 \Rightarrow \sqrt{x+a} - 7 = \sqrt{x-b} \Rightarrow \sqrt{x+a} > \sqrt{x-b} + 7$ $\Rightarrow \cancel{\sqrt{x-b} + 7} \sqrt{x-b} + 7 + \sqrt{x-b} - 7 = 7\sqrt{x-b}$	10