B دهم پسر   تکلیف ۲۱   سروش احوان

۲۰

Subject .   Year.   Month.   Day.

۱- $(1!+3!+5!+\dots+25!)(2!+4!+\dots+24!)$

فقط ۱! و ۳! در وجود عوامل ۲ و ۵   فقط ۲! و ۴! در وجود عوامل ۲ و ۵

$1!+3! = 7 \to$ یکان $2!+4! = 26 \to$ یکان ۶

$6\times 7 = 42 \Rightarrow$ یکان $\Rightarrow 2$

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۲- الف) $\sqrt[4]{(4+\sqrt7)^{-1}} \times \sqrt{1+\sqrt7} = \sqrt[4]{\frac{1}{4+\sqrt7}} \cdot \sqrt[4]{1+7+2\sqrt7} = \sqrt[4]{\frac{8+2\sqrt7}{4+\sqrt7}} = \sqrt[4]{\frac{2\cancel{(4+\sqrt7)}}{\cancel{4+\sqrt7}}}$

$= \sqrt[4]{2}$

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ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) = \frac{\sqrt2+\sqrt5}{(\sqrt2)\cancel{(\sqrt5+\sqrt2)}}\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)$

$\times\left(\sqrt{3-\sqrt5}+\sqrt{3+\sqrt5}\right)$

$\Longrightarrow \frac{\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)\left(\sqrt{3-\sqrt5}+\sqrt{3+\sqrt5}\right)}{\sqrt2\left(\sqrt{3-\sqrt5}+\sqrt{3+\sqrt5}\right)} = \frac{3-\sqrt5-3-\sqrt5}{\sqrt{6-2\sqrt5}+\sqrt{6+2\sqrt5}}$

$(\sqrt5-1)+(\sqrt5+1)$

$= \frac{-2\sqrt5}{2\sqrt5} = -1$

۳- الف) $\frac{\sqrt8+\sqrt{27}}{5-\sqrt6} - 2\left(\sqrt[4]{9}-1\right)^{-1} = \frac{2\sqrt2+3\sqrt3}{5-\sqrt6} - \frac{2}{\sqrt3-1} =$

$\frac{(2\sqrt2+3\sqrt3)}{5-\sqrt6}\cdot\frac{(\sqrt3+\sqrt2)}{(\sqrt2+\sqrt3)} - \frac{2}{\sqrt3-1}\cdot\frac{(\sqrt3+1)}{(\sqrt3+1)} = \frac{\cancel{(2\sqrt6+3\sqrt3)}(\sqrt2+\sqrt3)}{\cancel{5\sqrt6+3\sqrt3}} -$

$\frac{\cancel{2}(\sqrt3+1)}{\cancel{2}} = \sqrt3-\sqrt3+\sqrt2-1 = \sqrt2-1$

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ب) $\frac{\sqrt{27}-1}{4+\sqrt3} + (2-\sqrt3)^{-1} = \frac{(3\sqrt3-1)}{(4+\sqrt3)}\cdot\frac{(4-\sqrt3)}{(4-\sqrt3)} + \frac{1}{2-\sqrt3}\times\frac{(2+\sqrt3)}{(2+\sqrt3)} =$

$\frac{12\sqrt3-9-4+\sqrt3}{13} + \frac{(2+\sqrt3)}{1} = \frac{13(\sqrt3-1)}{13} + 2+\sqrt3 = 2\sqrt3+1$

۴- الف) $\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}} - 2 = \sqrt{\frac{\left(\sqrt{\sqrt3+1}+\sqrt{\sqrt3-1}\right)^2}{\left(\sqrt{\sqrt3-\sqrt2}\right)^2}} - 2 = \sqrt{\frac{\sqrt3+1+\sqrt3-1+2\sqrt2}{\sqrt3-\sqrt2}} - 2$

$= \sqrt{\frac{(2\sqrt3+2\sqrt2)}{\sqrt3-\sqrt2}\cdot\frac{(\sqrt3+\sqrt2)}{\sqrt3+\sqrt2}} - 2 = \sqrt{\frac{2(\sqrt3+\sqrt2)^2}{3-2}} - 2 = \sqrt6+2-2 = \sqrt6$

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۴- ب) $\sqrt[3]{\sqrt{2}\cdot 8} \cdot \sqrt[4]{162} \cdot \sqrt{4\sqrt[6]{3}} = \sqrt[3]{4}\cdot 3\sqrt[4]{2}\cdot \sqrt[12]{2^{13}}$ ۲

$= \sqrt[12]{2^8\cdot 3^{11}\cdot 2^3\cdot 2^{13}} = \sqrt[12]{2^{24}\cdot 3^{12}} = 2^2\cdot 3 = \underline{12}$

۵- $\dfrac{3^x(1+3+9+27+81+243)}{2^x(\frac{1}{4}+\frac{1}{2}+1+2+4+8)} = 52 \Rightarrow \dfrac{3^x(364)}{2^x(\frac{63}{4})} = 52$ ۲

$\Rightarrow 819\times 2^x = 3^x \times 364 \Rightarrow \left(\frac{3}{2}\right)^x = \frac{819}{364} = 2.25 = (1.5)^2 \Rightarrow \underline{x=2}$

۶- $a = \sqrt[4]{\sqrt{5}-2}\ ,\ b = \sqrt[4]{\sqrt{5}+2} \qquad (a^2+b^2+2ab)^2(a^2+b^2-2ab)^2 = t(3-\sqrt{5})$

$= (a^4+b^4+\underbrace{2a^2b^2-4a^2b^2}_{-2a^2b^2})^2 = (\sqrt{5}-2+\sqrt{5}+2-2\sqrt{1})^2 = (2\sqrt{5}(\sqrt{5}-1))^2$ ۲

$= 8(5+1-2\sqrt{5}) = 16(3-\sqrt{5}) = t(3-\sqrt{5}) \Rightarrow \underline{t = 16}$

۷- $a = \sqrt[4]{7-4\sqrt{3}} = \sqrt[4]{(2-\sqrt{3})^2} = \sqrt{2-\sqrt{3}} \qquad \left(a+\frac{1}{a}+\sqrt{2}\right)^2\left(a+\frac{1}{a}-\sqrt{2}\right)^2$

$= \left(a^2+\frac{1}{a^2}+2-2\right)^2 = \left(a^2+\frac{1}{a^2}\right)^2 = a^4+\frac{1}{a^4}+2 \Rightarrow 7-4\sqrt{3}+\frac{1}{7-4\sqrt{3}}+2$

$= \dfrac{97-56\sqrt{3}-8\sqrt{3}+14+1}{7-4\sqrt{3}} = \dfrac{112-64\sqrt{3}}{7-4\sqrt{3}} = \dfrac{16(\cancel{7-4\sqrt{3}})}{\cancel{7-4\sqrt{3}}}$ ۲

$= 16 = 2^t \Rightarrow \underline{t=4}$

۸- $\sqrt[15]{2^{10}}\cdot 2^{\frac{4}{3}} = 2^{\frac{2}{3}}\cdot 2^{\frac{4}{3}} = 2^{\frac{6}{3}} = 2^2 = \underline{4}$ ۲

$8^{-\frac{1}{3}} = (2^3)^{-\frac{1}{3}} = 2^{-1} = \underline{\frac{1}{2}}$

۹- $\sqrt[7]{a} = 27a^{\frac{15}{7}} \Rightarrow \sqrt[7]{a} = 27\sqrt[7]{a^{15}} = 27a^2\sqrt[7]{a}$

$\Rightarrow \sqrt[7]{a} = 27a^2\sqrt[7]{a} \Rightarrow 27a^2 = 1 \Rightarrow a = \dfrac{1}{3\sqrt{3}}$ ۲

$\dfrac{\frac{1}{a}-3}{1+\sqrt{3}} = \dfrac{3\sqrt{3}-3}{1+\sqrt{3}} = \dfrac{3(\sqrt{3}-1)}{(1+\sqrt{3})}\cdot\dfrac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \dfrac{3(3+1-2\sqrt{3})}{2}$

$= \dfrac{6(2-\sqrt{3})}{2} = \underline{6-3\sqrt{3}}$

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$-)$.

$$(\sqrt{x+a}-\sqrt{x-۴})(\sqrt{x+a}+\sqrt{x-۴}) = ۲(\sqrt{x+a}+\sqrt{x-۴})$$

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$$\Rightarrow \cancel{x}+a-\cancel{x}+۴ = ۲(\sqrt{x+a}+\sqrt{x-۴}) \Rightarrow a+۴ = ۲(\sqrt{x+a}+\sqrt{x-۴})$$

$$\Rightarrow \sqrt{x+a}+\sqrt{x-۴} = \frac{a}{۲}+۲ \quad , \quad \frac{a}{۲}+۲-۲ = \frac{a}{۲}$$