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نام و نام خانوادگی ......پرنیان یزدی...... پاسخنامه تشریحی تکلیف شماره ۱...۲... کلاس ....دهم........

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از ۵! به بعد یکان‌ها صفر است پس قبل آن را حساب می‌کنیم!

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$1! + 3! + 2! + 4! = 33$

$1 + 6 + 2 + 24$

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$1!+3! \to 1+6=7$

$2!+4! \to 2+24=26$ → یکان: $7\times 6 = 42$ → یکان $= 2$

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الف) ۱

$\frac{\sqrt{1+\sqrt7}}{\sqrt[4]{4+\sqrt7}} \xrightarrow{\text{به توان ۴}} \frac{1+7+2\sqrt7}{4+\sqrt7} = \sqrt[4]{2}$

ب)

$\underbrace{\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)}_{a}\underbrace{\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)}_{b} \to a = \frac{(\sqrt2+\sqrt5)}{\sqrt2(\sqrt2+\sqrt5)} = \frac{1}{\sqrt2}$

$b^2 = \left((3-\sqrt5)+(3+\sqrt5)-2\sqrt{9-5}\right) = 6-4 = 2 \to b^2=2 \to b = \begin{cases} +\sqrt2 & ✗ \\ -\sqrt2 & ✓ \end{cases}$

$\frac{\sqrt{6-2\sqrt5} - \sqrt{6+2\sqrt5} + \sqrt{16-5\sqrt5} - \sqrt{16+5\sqrt5}}{\sqrt{10}+2}$

$ab = \frac{1}{\sqrt2}\times(-\sqrt2) = -1$

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الف) ۰

$\frac{2\sqrt3+3\sqrt2}{5-\sqrt6} - \frac{2}{\sqrt3-1} \to \frac{3\sqrt6-1}{5\sqrt3-\sqrt{18}-5+\sqrt6}$

$8\sqrt3-9$

(پایین صفحه)

ب)

$\frac{\sqrt{27}-1}{4+\sqrt3} + \frac{1}{2-\sqrt3} \to \frac{\overset{6\sqrt3}{2\sqrt{27}} - 2 - \overset{-9}{\sqrt{81}} + \sqrt3}{5-2\sqrt3} \xrightarrow{\text{به توان ۲}} \frac{36\times3-49}{25-12} = 11$

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الف)

$\xrightarrow{\text{به توان ۲}} \frac{\cancel{2}+\sqrt3+\sqrt4-2-2\sqrt3-1}{\sqrt3-\sqrt2} - 2 \to \frac{2(\sqrt3-\sqrt2)}{\cancel{\sqrt3-\sqrt2}} - 2 = 0$

۱ (پایین صفحه)

ب)

$\xrightarrow{\text{به توان ۴}} \sqrt[3]{2^8} \times 162 \times 16\sqrt[3]{2} = \sqrt[4]{10368} = 12$

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$\frac{3^n(3+9+27+81+243)}{2^n(2^{-2}+2^{-1}+1+2+2^2+2^3)} \to \left(\frac32\right)^n \times \frac{63}{4} = 52 \to \left(\frac32\right)^n = \frac{68}{21}$

$1+3+3^2+\dots+3^5 \to q=3,\ a_1=1$ دنباله هندسی $\to S_n = a_1\frac{(1-q^n)}{1-q} \to S_n = 1\times\frac{(1-3^6)}{1-3} = 364$

$1+2+2^2+\dots+2^5 \to q=2,\ a_1=1$ دنباله هندسی $\to S_n = 1\times\frac{(1-2^6)}{(1-2)} = 63$

$\frac{3^n(1+3+3^2+\dots+3^5)}{2^{n-2}(1+2+2^2+\dots+2^5)} \to \frac{3^n(364)}{2^n\times2^{-2}\times63} = 52 \to \left(\frac32\right)^n \times \frac{4\times364}{63} = 52 \to \left(\frac32\right)^n = \frac94 \to n=2$

$((a-b)^2)^2((a+b)^2)^2=(a^2-b^2)^4=\left(\sqrt{\sqrt6-2}-\sqrt{\sqrt6+2}\right)^4=\left(\sqrt6-2+\sqrt6+2-2\sqrt{(\sqrt6-2)(\sqrt6+2)}\right)^2=$

$(2\sqrt6-2\sqrt2)^2=4(6+2-2\sqrt{12})=4(8-4\sqrt3)=16(2-\sqrt3)\rightarrow t=16$

$(a-b)^2(a+b)^2=((a-b)(a+b))^2\quad (a^2-b^2)^2\quad a^4+b^4-2a^2b^2=$ 1 $\sqrt6-2+\sqrt6+2-2\left(\sqrt[4]{\sqrt6-2}\right)^2\left(\sqrt[4]{\sqrt6+2}\right)^2\quad 2\sqrt6-2\sqrt[4]{(\sqrt6-2)^2}$ $\sqrt[4]{(\sqrt6+2)^2}=t(2-\sqrt3)\quad 2\sqrt6-2\sqrt[4]{((\sqrt6-2)(\sqrt6+2))^2}=2\sqrt6-2\sqrt2=$ $2(\sqrt6-\sqrt2)=t(2-\sqrt3)\quad t=\frac{2(\sqrt6-\sqrt2)}{2-\sqrt3}\quad t=\frac{2(\sqrt2\sqrt3-\sqrt2)}{2-\sqrt3}=$ $\frac{2\sqrt2(\sqrt3-1)}{2-\sqrt3}$ ✓	۶
$\left((a+\frac1a)^2-2\right)^2=(a^2+\frac1{a^2}+2-2)^2=(a^2+\frac1{a^2})^2=a^4+\frac1{a^4}+2$ 1 $\left(\left((a+\frac1a)+\sqrt2\right)\left((a+\frac1a)-\sqrt2\right)\right)^2=2^t\quad 2^4=2^t\rightarrow t=4$ $\left((a+\frac1a)^2-2\right)^2=2^t\quad (a^2+\frac1{a^2}+2-2)^2=2^t\quad (a^2+\frac1{a^2})^2=2^t$ $=7-4\sqrt3+\frac{1}{7-4\sqrt3}+2=9-4\sqrt3+\left(\frac{1}{7-4\sqrt3}\times\frac{7+4\sqrt3}{7+4\sqrt3}\right)=9-4\sqrt3+\frac{7+4\sqrt3}{49-48}=16$	۷
$A=\sqrt[5]{2^2\sqrt[3]{2^3\times2}}\,(2)^{\frac43}=\sqrt[5]{2^2\times2\sqrt[3]{2}}\,(2)^{\frac43}=\sqrt[5]{2^3\sqrt[3]{2}}\,(2)^{\frac43}=$ 4 $\sqrt[5]{2^{\frac93}\,2^{\frac13}}\,(2)^{\frac43}=\sqrt[5]{2^{\frac{10}{3}}}\,(2)^{\frac43}=(2)^{\frac23}(2)^{\frac43}=2^2=4\rightarrow \boxed{A=4}$ $(2A)^{-\frac13}=(8)^{-\frac13}=\left(\frac18\right)^{\frac13}=\sqrt[3]{\frac18}=\sqrt[3]{\frac{1}{2^3}}=\frac12$ ✓	۸
$\sqrt[7]{a}=3^3a^{\frac{15}{7}}\rightsquigarrow a=3^{21}\times a^{15}\rightsquigarrow a^{-14}=3^{21}\rightsquigarrow a^{-1}=3\sqrt3$ $\frac{3\sqrt3-3}{1+\sqrt3}\rightsquigarrow\frac{3(\sqrt3-1)}{1+\sqrt3}\times\frac{\sqrt3-1}{\sqrt3-1}\quad\frac{3(4-2\sqrt3)}{3-1}=6-3\sqrt3$ ✓ 4	۹
$\sqrt{x+9}+\sqrt{x-4}=2+2\sqrt{x-4}$ ۰ $2+2\sqrt{x-4}-2=2\sqrt{x-4}$ ✓ $\underbrace{(\sqrt{x+a}+\sqrt{x-4})}\underbrace{(\sqrt{x+a}-\sqrt{x-4})}_{2}=(x+a)-(x-4)=a+4$ $\rightarrow\frac a2+2=\frac{a+4}{2}$	۱۰

$\sqrt{x+a}+\sqrt{x-4}=\frac a2+2\rightarrow\sqrt{x+a}+\sqrt{x-4}-2=\frac a2+2-2=\frac a2$

سوال ۳)

الف) $\frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{6}} - 2\left(\sqrt[4]{9}-1\right)^{-1}$

$$\frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{6}} = \frac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}} \times \frac{5+\sqrt{6}}{5+\sqrt{6}} = \frac{10\sqrt{2}+4\sqrt{3}+15\sqrt{3}+9\sqrt{2}}{19} = \frac{19\sqrt{2}+19\sqrt{3}}{19} = \sqrt{2}+\sqrt{3}$$

$$\left(\sqrt[4]{9}-1\right)^{-1} = \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{1}{2}\left(\sqrt{3}+1\right) \longrightarrow \left(\sqrt{2}+\sqrt{3}\right) - 2\left(\frac{1}{2}\left(\sqrt{3}+1\right)\right) = \sqrt{2}-1$$

ب) $\frac{\sqrt{27}-1}{4+\sqrt{3}} + \left(2-\sqrt{3}\right)^{-1}$

$$\frac{\sqrt{27}-1}{4+\sqrt{3}} = \frac{3\sqrt{3}-1}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{12\sqrt{3}-9-4+\sqrt{3}}{13} = \frac{13\sqrt{3}-13}{13} = \sqrt{3}-1$$

$$\left(2-\sqrt{3}\right)^{-1} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2+\sqrt{3} \longrightarrow \sqrt{3}-1+2+\sqrt{3} = 2\sqrt{3}+1$$

سوال ۴)

الف) $\frac{\overbrace{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}^{A}}{\sqrt{\sqrt{3}-\sqrt{2}}} - 2 \longrightarrow \sqrt{6}+2-2 = \sqrt{6}$

$$A^2 = \left(\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}\right)^2 = 1+\sqrt{3}+\sqrt{3}-1+2\sqrt{3-1} = 2\sqrt{3}+2\sqrt{2} \longrightarrow A = \sqrt{2}\left(\sqrt{\sqrt{3}+\sqrt{2}}\right)$$

$$\frac{\sqrt{2}\left(\sqrt{\sqrt{3}+\sqrt{2}}\right)}{\left(\sqrt{\sqrt{3}-\sqrt{2}}\right)} = \sqrt{2} \times \sqrt{\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}} \longrightarrow \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \left(\sqrt{3}+\sqrt{2}\right)^2 \longrightarrow \sqrt{2} \times \sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2} = \sqrt{2}\left(\sqrt{3}+\sqrt{2}\right)$$