۱- $(1+6+0+\cdots+0)\times(2+2^6+0+\cdots+0)=7\times 6=42$ ✓

۲- الف) $\sqrt[6]{(4+\sqrt{7})^{-1}}\sqrt{1+\sqrt{7}} \Rightarrow (4+\sqrt{7})^{-\frac{1}{6}}(1+\sqrt{7})^{\frac{1}{2}} \Rightarrow \frac{(1+\sqrt{7})^{\frac{1}{2}}}{(4+\sqrt{7})^{\frac{1}{6}}}=\left(\frac{(1+\sqrt{7})^3}{4+\sqrt{7}}\right)^{\frac{1}{6}}$

$\Rightarrow 2^{\frac{1}{6}}=\sqrt[6]{2}$

ب) $\sqrt{3+\sqrt{5}}=\frac{\sqrt{5}+1}{\sqrt{2}} \Rightarrow \frac{\sqrt{5}-1}{\sqrt{2}}-\frac{\sqrt{5}+1}{\sqrt{2}}=-\sqrt{2} \Rightarrow \frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2}\times(-\sqrt{2})=-1$

۳- الف) $\frac{\sqrt{8}+\sqrt{24}}{5-\sqrt{6}}-2\left(\sqrt{9}-1\right)^{-1} \Rightarrow \frac{2\sqrt{2}+2\sqrt{3}}{5-\sqrt{6}}-\frac{2}{2-1} \Rightarrow \frac{2\sqrt{2}+2\sqrt{3}}{5-\sqrt{6}}\times\frac{5+\sqrt{6}}{5+\sqrt{6}}=\frac{19(\sqrt{2}+\sqrt{3})}{19}=\sqrt{2}+\sqrt{3}$

$\Rightarrow \sqrt{2}+\sqrt{3}-\sqrt{3}+1=\sqrt{2}-1$

ب) $\frac{2\sqrt{3}-1}{\sqrt{3}+2}+\frac{1}{2-\sqrt{3}} \Rightarrow \frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{2+\sqrt{3}}{1} \Rightarrow \frac{2\sqrt{3}-1}{\sqrt{3}+2}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=1-\sqrt{3}$

$\Rightarrow 2+\sqrt{3}+1-\sqrt{3}\Rightarrow 3$

۴- الف) $\left(\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}\right)^2=1+\sqrt{3}+\sqrt{3}-1+2\sqrt{(1+\sqrt{3})(\sqrt{3}-1)}=2\sqrt{3}+2\sqrt{2} \Rightarrow \sqrt{2\sqrt{3}+2\sqrt{2}}=\sqrt{2}\sqrt{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \sqrt{\sqrt{3}-\sqrt{2}}=\frac{1}{\sqrt{\sqrt{3}+\sqrt{2}}} \Rightarrow \sqrt{2}\sqrt{\sqrt{3}+\sqrt{2}}\times\sqrt{\sqrt{3}+\sqrt{2}}=\sqrt{2}\left(\sqrt{3}+\sqrt{2}\right)=\sqrt{6}+2 \Rightarrow \sqrt{6}+2-2=\sqrt{6}$

ب) $\sqrt[4]{\sqrt{3}\sqrt{4^8}}=4^{\frac{8}{12}} \quad \sqrt[6]{12^4}=2^{\frac{4}{6}}\times 3^{\frac{4}{6}} \quad \sqrt[4]{2\sqrt[6]{4^3}}=2^{\frac{12}{12}}$

$\Rightarrow 2^{\frac{12}{12}}\times 3^{\frac{4}{6}}\times 2^{\frac{4}{6}}\times 2^{\frac{8}{12}} \Rightarrow 2^2\times 3=12$

۵- $\frac{3^{2x}\left(1+3+3^2+\cdots+3^5\right)}{2^{2x}\left(2^{-2}+2^{-1}+\cdots+2^3\right)} \Rightarrow \frac{3^{2x}}{2^{2x}}\times 364\times\frac{4}{63}=52 \Rightarrow \frac{3^{2x}}{2^{2x}}\times 364\times\frac{4}{63}\times\frac{1}{52}=1 \Rightarrow \frac{3^{2x}}{2^{2x}}\times\frac{4}{9}=1$

$\Rightarrow x=2$

۶- $\left((a-b)^2\right)^2\left((a+b)^2\right)^2 = (a-b)^4(a+b)^4 \Rightarrow (a^2-b^2)^4 \Rightarrow a^2-b^2 = \sqrt{\sqrt{6}-2} - \sqrt{\sqrt{6}+2}$

$\Rightarrow (\sqrt{6}-2)+(\sqrt{6}+2) = 2\sqrt{6}$, $(\sqrt{6}-2)(\sqrt{6}+2) = 2 \Rightarrow (2\sqrt{6}-2\sqrt{2})^2 = 24+8-8\sqrt{12} = 32-16\sqrt{3}$

$\Rightarrow 32-16\sqrt{3} = 16(2-\sqrt{3}) \Rightarrow \boxed{t=16}$

۷- $\left((a+\frac{1}{a})^2-2\right)^2 \Rightarrow a^2+\frac{1}{a^2}+2-2 \Rightarrow \left(a^2+\frac{1}{a^2}\right)^2 \Rightarrow \sqrt{7-4\sqrt{3}} = \sqrt{(2-\sqrt{3})^2} \Rightarrow a^2 = 2-\sqrt{3}$

$\Rightarrow \frac{1}{a^2} = 2+\sqrt{3} \Rightarrow a^2+\frac{1}{a^2} = (2-\sqrt{3})+(2+\sqrt{3}) = 4 \Rightarrow 4^2 = 16 = 2^4 \Rightarrow \boxed{k=4}$

★ $(x+y)^2(x-y)^2(x^2-y^2)^2$

۸- $2^{\frac{3}{4}} \times 2^{\frac{2}{3}} = 2^{\frac{10}{3}} \Rightarrow 2^{\frac{10}{3}} \times \frac{1}{2} = 2^{\frac{10}{15}} = \sqrt[5]{2^{\sqrt[3]{16}}}$

$\left(\frac{1}{2}\right)^{-\frac{3}{4}} = (2^{-1})^{-\frac{3}{4}} \Rightarrow 2^{\frac{10}{15}} \times 2^{\frac{3}{4}} = 2^{\frac{30}{15}} = 4$

$(2A)^{-\frac{1}{2}} = (2\times 4)^{-\frac{1}{2}} \Rightarrow \frac{1}{\sqrt{8}} = \frac{1}{2}$

۹- $a^{\frac{1}{2}} = 27 \times a^{\frac{5}{2}} \Rightarrow a = 3^{\frac{21}{2}} \times a^{\frac{21}{2}} \Rightarrow a = 243a^3 \Rightarrow 243a^3 - a = 0 \Rightarrow a(243a^2-1) = 0 \Rightarrow a \overset{\circ}{\circlearrowleft} -\frac{1}{\sqrt{243}}, \frac{1}{\sqrt{243}}$ ✓

$\Rightarrow \frac{\sqrt{3}-3}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = -2\sqrt{3}+3$

۱۰- $A = \sqrt{x+5} \quad B = \sqrt{x-3} \quad A-B = 2 \Rightarrow A-B = ? \Rightarrow (A-B)(A+B) = A^2-B^2$

$\Rightarrow 2(A+B) \Rightarrow (x+5)-(x-3) = 5+3 \Rightarrow 2(A+B) = 5+3 \Rightarrow A+B = \frac{5+3}{2}$

$\Rightarrow A+B-2 = \frac{(5+3)}{2} - 2 \Rightarrow \frac{(5+3-4)}{2} = \boxed{\frac{5}{2}}$