14,5

نام و نام خانوادگی ...امیرمحمدی... پاسخنامه تشریحی تکلیف شماره ..۲۱.. کلاس ...B ریاضی...

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$$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+26!)$$

$$(1+6+0+0+\dots+0)(2+4+0+\dots+0)$$

$$7 \times 6 = 42$$

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$$\sqrt[4]{(4+\sqrt7)^{-1}}\sqrt{1+\sqrt7} = \sqrt[4]{\frac{1}{4+\sqrt7}}\times\sqrt[4]{(1+\sqrt7)^2} = \sqrt[4]{\frac{8+2\sqrt7}{4+\sqrt7}} = \sqrt[4]{2}$$

$$\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) = \frac{1}{\sqrt2}\times\frac{\sqrt5-1-\sqrt5-1}{\sqrt2} = \frac{-2}{2} = -1$$

$$\frac{\sqrt2+\sqrt5}{\sqrt2(\sqrt5+\sqrt2)} \qquad \frac{\sqrt{6-2\sqrt5}-\sqrt{6+2\sqrt5}}{\sqrt2}$$

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$$\frac{\sqrt8+\sqrt{27}}{5-\sqrt6} - 2\left(\sqrt[4]{\frac{\sqrt3}{\sqrt9}}-1\right)^{-1} = \frac{(\sqrt8+\sqrt{27})(5+\sqrt6)}{\cancel{25-6}_{\,19}} - \frac{\cancel{2}(\sqrt3+1)}{\cancel{2}} = \frac{5\sqrt8+5\sqrt{27}+\sqrt{48}+\sqrt{162}}{19} - \sqrt{}$$

$$= \frac{19(\sqrt3+\sqrt2)}{19} - \sqrt3 - 1 = \sqrt2 - 1$$

$$\frac{\sqrt{27}-1}{4+\sqrt3} + \frac{1}{2-\sqrt3} = \frac{(\sqrt{27}-1)(4-\sqrt3)}{13} + 2+\sqrt3 = \frac{12\sqrt3-9-4+\sqrt3}{13} + 2+\sqrt3$$

$$\frac{\cancel{13}(\sqrt3-1)}{\cancel{13}} + 2+\sqrt3 = 2\sqrt3+1$$

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$$A = \frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}} - 2 \Rightarrow A^2 = \frac{1+\sqrt3+\sqrt3-1+2\sqrt2}{\sqrt3-\sqrt2} = \frac{2(\sqrt3+\sqrt2)}{\sqrt3-\sqrt2} = 2(\sqrt3+\sqrt2)^2$$

$$A = \sqrt2(\sqrt3+$$

$$\sqrt6+2-2 = \boxed{\sqrt6}$$

$$\sqrt[4]{\sqrt[3]{\sqrt{2^8}}}\times\sqrt[4]{162}\times\sqrt{4\sqrt[4]{2}} =$$

$$\sqrt[12]{2^8}\times\sqrt[4]{81\times2}\times2\sqrt[12]{2} = 2^{\frac{8}{12}}\times3\times2^{\frac14}\times2\times2^{\frac{1}{12}} = 2\times3\times2 = \boxed{12}$$

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$$\frac{3^n(1+3+3^2+3^3+3^4+3^5)}{2^{n-2}(1+2+2^2+2^3+2^4+2^5)} = \frac{3^n(\cancel{364})}{2^{n-2}(\cancel{63})} = \cancel{81}$$

$$\Rightarrow \frac{3^n}{2^{n-2}\times3^2} = 1 \to \frac{3^{n-2}}{2^{n-2}} = 1 \Rightarrow \boxed{n=2}$$

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$a=\sqrt[۴]{\sqrt{۶}-۲} \rightarrow a^{۴}=\sqrt{۶}-۲$

$b=\sqrt[۴]{\sqrt{۶}+۲} \rightarrow b^{۴}=\sqrt{۶}+۲$

$\Rightarrow a^{۲}b^{۲}=\sqrt{\sqrt{۶}-۲}\times\sqrt{\sqrt{۶}+۲}=\sqrt{A}=۲\sqrt{۲}$

$\underbrace{(a^{۲}+b^{۲}-۲ab)}_{(a-b)^{۲}}{}^{۲}\underbrace{(a^{۲}+b^{۲}+۲ab)}_{(a+b)^{۲}}{}^{۲}=(a^{۲}-b^{۲})^{۴}=(a^{۴}+b^{۴}-۲a^{۲}b^{۲})^{۲}$

$=(\sqrt{۶}-۲+\sqrt{۶}+۲-۴)^{۲}=(۲\sqrt{۶}-۲\sqrt{۲})^{۲}$

$=۲۴+۸-۱۶\sqrt{۳}=۳۲-۱۶\sqrt{۳}=۱۶(۲-\sqrt{۳})\rightarrow t=۱۶$

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$\left(a+\frac{۱}{a}+\sqrt{۲}\right)^{۲}\left(a+\frac{۱}{a}-\sqrt{۲}\right)^{۲}=۲\Rightarrow\left(\left(a+\frac{۱}{a}\right)^{۲}-۲\right)^{۲}=۲$

$\rightarrow\left(a^{۲}+\frac{۱}{a^{۲}}\right)^{۲}=۲^{t}\rightarrow a^{۴}+\frac{۱}{a^{۴}}+۲=۲^{t}\rightarrow ۷-۴\sqrt{۳}+\frac{۱}{۷-۴\sqrt{۳}}+۲=۲^{t}$

$۷-۴\sqrt{۳}+\frac{۷+۴\sqrt{۳}}{\cancel{۴۹-۴۸}}+۲=۲^{t}\Rightarrow ۱۶=۲^{t}\Rightarrow \boxed{t=۴}$

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$A=\sqrt[۵]{۴^{۳}\sqrt{۱۶}\left(\frac{۱}{۲}\right)^{\frac{-۴}{۳}}}=\sqrt[۱۵]{۲^{۱۰}}\times\sqrt[۳]{۲^{۴}}=۲^{\frac{۲}{۳}}\times ۲^{\frac{۴}{۳}}=۴$

$(۲A)^{\frac{-۱}{۳}}=۸^{\frac{-۱}{۳}}=\frac{۱}{\sqrt[۳]{۸}}=\boxed{\frac{۱}{۲}}$

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$\sqrt[۷]{a}=۲۷a^{\frac{۱۵}{۷}}\rightarrow a^{\frac{۱}{۷}}=۲۷a^{\frac{۱۵}{۷}}\rightarrow ۱=۲۷a^{۲}\rightarrow a^{۲}=\frac{۱}{۲۷}=\frac{۱}{۳\sqrt{۳}}$

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$\left(\frac{۱}{a}-۳\right)=۳\sqrt{۳}-۳=۳(\sqrt{۳}+۱)$

۳ برابر

$۳(۲-\sqrt{۳})=۶-۳\sqrt{۳}$

$\rightarrow\left(\frac{۱}{a}-۳\right)=۳\sqrt{۳}-۳=۳(\sqrt{۳}-۱)\rightarrow\frac{۳(\sqrt{۳}-۱)}{\sqrt{۳}+۱}\times\frac{\sqrt{۳}-۱}{\sqrt{۳}-۱}=\frac{۳(۱+۳-۲\sqrt{۳})}{۳-۱}=\frac{۳(۴-۲\sqrt{۳})}{۲}=$

۱۰

$\sqrt{x+a}-\sqrt{x-۴}=۲\rightarrow x+a-x+۴=۲(\sqrt{x+a}+\sqrt{x-۴})$

$\rightarrow a+۴=۲(\sqrt{x+a}+\sqrt{x-۴})\rightarrow\sqrt{x+a}+\sqrt{x-۴}=\frac{a+۴}{۲}$

$\Rightarrow\sqrt{x+a}+\sqrt{x-۴}-۲=\frac{a+۴}{۲}-۲=\boxed{\frac{a}{۲}}$

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