

$$\frac{(1+9+25+\dots+100)}{\sqrt{\quad}} \cdot \frac{(1+4+9+\dots+100)}{\sqrt{\quad}}$$

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الف) $\sqrt[3]{(1+\sqrt{5})^{-1}} \times \sqrt{1+\sqrt{5}} = \sqrt[3]{\frac{(1+\sqrt{5})^2}{1+\sqrt{5}}} = \sqrt[3]{2} \times (1+\sqrt{5})^2 = 1 + \sqrt{5} + 2\sqrt{5} = 1 + 3\sqrt{5} = 2(1+\sqrt{5})$

ب) $\frac{\sqrt{2+\sqrt{5}}}{\sqrt{1+2}} = \frac{\sqrt{2}}{\sqrt{3}} \rightarrow \frac{\sqrt{2}}{\sqrt{3}} \times (\sqrt{2-\sqrt{5}} - \sqrt{2+\sqrt{5}}) = \frac{1}{\sqrt{3}} (\sqrt{4-2\sqrt{5}} - \sqrt{4+2\sqrt{5}})$

$= \frac{1}{\sqrt{3}} (\sqrt{5-1} - \sqrt{5+1}) = \frac{1}{\sqrt{3}} (\sqrt{4} - \sqrt{6}) = \frac{2-\sqrt{6}}{\sqrt{3}} = -1$ ✓

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الف) $\frac{\sqrt{11}+\sqrt{7}}{8-\sqrt{6}} - 2(\sqrt{6}-1) = \frac{11\sqrt{2}+9\sqrt{2}}{19} - \sqrt{2} - 1 = \frac{11\sqrt{2}-19}{19}$

ب) $\frac{\sqrt{11}-1}{2\sqrt{3}} + (2-\sqrt{3})^{-1}$

$\frac{\sqrt{11}-1}{2\sqrt{3}} = \frac{2\sqrt{3}-1}{2\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{4\sqrt{3}-9-2+\sqrt{3}}{13} = \frac{5\sqrt{3}-11}{13}$

$(2-\sqrt{3})^{-1} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$

$\frac{5\sqrt{3}-11}{13} + 2+\sqrt{3} = \frac{5\sqrt{3}-11+26+13\sqrt{3}}{13} = \frac{18\sqrt{3}+15}{13} = \frac{3(6\sqrt{3}+5)}{13}$

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الف) $(\sqrt{1+\sqrt{2}} + \sqrt{2}-1)^2 = 1 + \sqrt{2} + \sqrt{2}-1 + 2\sqrt{(1+\sqrt{2})(2-\sqrt{2})} = 2\sqrt{2} + 2\sqrt{2} = 4\sqrt{2}$

$\sqrt{4\sqrt{2}} = \sqrt{2} \sqrt{2\sqrt{2}} = \sqrt{2} \sqrt{2} \sqrt{2} = 2\sqrt{2}$

ب) $\sqrt{2-\sqrt{2}} = \frac{1}{\sqrt{2+\sqrt{2}}} \Rightarrow \sqrt{2} \times \sqrt{2+\sqrt{2}}$

$\times \sqrt{2-\sqrt{2}} = \sqrt{2} (\sqrt{2+\sqrt{2}}) = \sqrt{4+2\sqrt{2}} = \sqrt{6}$ ✓

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$\frac{3^{91} (1+9+27+81+243)}{3^{90} (1+3+9+27+81)} = \frac{3^{91} \times 320}{3^{90} \times 121} = \frac{3^{91} \times 320}{3^{90} \times 121}$

$\frac{3^{91} \times 320}{3^{90} \times 121} = 3 \times \frac{320}{121} = 3 \times \frac{320}{121}$

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$(a-b)^r (a+b)^r = (a-b)^r (a+b)^r \Rightarrow (a^r-b^r)^{\frac{1}{r}} = a^r-b^r = \sqrt[r]{a^r-b^r} \cdot \sqrt[r]{a^r-b^r}$ $\Rightarrow \sqrt[r]{a^r-b^r} = \sqrt[r]{a^r-b^r} + \sqrt[r]{a^r-b^r} = 2\sqrt[r]{a^r-b^r} \quad (\sqrt[r]{a^r-b^r})(\sqrt[r]{a^r-b^r}) = 2 \Rightarrow (\sqrt[r]{a^r-b^r})^r = 2^r \Rightarrow \sqrt[r]{a^r-b^r} = 2$ $\Rightarrow (2^r-1)\sqrt[r]{a^r-b^r} = 14 \Rightarrow (2^r-1) = 14 \Rightarrow 2^r = 15 \Rightarrow r = 4$	6
$a^{\frac{1}{r}} + \frac{1}{a^{\frac{1}{r}}} = 2 \Rightarrow (a^{\frac{1}{r}} + \frac{1}{a^{\frac{1}{r}}})^r = 2^r \Rightarrow \sqrt[r]{a^{\frac{1}{r}} + \frac{1}{a^{\frac{1}{r}}}} = \sqrt[r]{2^r} \Rightarrow a^{\frac{1}{r}} = 2$ $\Rightarrow a^{\frac{1}{r}} + \frac{1}{a^{\frac{1}{r}}} = 2 + \frac{1}{2} = \frac{5}{2} = 2.5 \Rightarrow 2.5 = 2 \Rightarrow r = 2$	7
$A = \sqrt[r]{\frac{1}{r}} \times \left(\frac{1}{r}\right)^{-\frac{1}{r}} = r^{\frac{1}{r}} \times r^{\frac{1}{r}} \times r^{\frac{1}{r}} = r^{\frac{3}{r}} = 2$ $(rA)^{\frac{1}{r}} = \frac{1}{r} = \frac{1}{r}$ $\Rightarrow \frac{1}{\sqrt[r]{r}} = \frac{1}{r}$	8
$a^{\frac{1}{r}} = r \times a^{\frac{1}{r}} \Rightarrow a = r^{\frac{1}{r}} \times a^{\frac{1}{r}} \Rightarrow a = r a^{\frac{1}{r}} \Rightarrow (a^{\frac{1}{r}} - a)^{\frac{1}{r}} = \frac{r(\sqrt[r]{r}-1)}{\sqrt[r]{r}+1} \times \frac{\sqrt[r]{r}-1}{\sqrt[r]{r}-1} = \frac{r(1+\sqrt[r]{r}-r\sqrt[r]{r})}{r-1} = \frac{r(1+\sqrt[r]{r})}{r-1}$ $\Rightarrow a(r a^{\frac{1}{r}} - 1) = 0 \Rightarrow \begin{cases} a = 0 \\ a = -\frac{1}{r} \\ a = \frac{1}{r} \end{cases}$ $a^{\frac{1}{r}} = \frac{1}{r} \Rightarrow a = \frac{1}{r^{\frac{1}{r}}} = \frac{1}{\sqrt[r]{r}}$ $\frac{\sqrt[r]{r}-r}{\sqrt[r]{r}+1} \times \frac{\sqrt[r]{r}-1}{\sqrt[r]{r}-1} = \frac{-r\sqrt[r]{r}+r}{r-1} = -\sqrt[r]{r}+1$ $r(1-\sqrt[r]{r}) = r - r\sqrt[r]{r}$	9
$\sqrt{x+a} + \sqrt{x-a} = 2 \Rightarrow \sqrt{x+a} + \sqrt{x-a} = 2 \Rightarrow \sqrt{x+a} - 2 = -\sqrt{x-a}$ $\sqrt{x+a} - 2 = -\sqrt{x-a} \Rightarrow (\sqrt{x+a} + \sqrt{x-a})(\sqrt{x+a} - \sqrt{x-a}) = (2-a) - (x-a) = a+x$ $\frac{a}{r} + r \leq \frac{a+r}{r}$ $\sqrt{x+a} + \sqrt{x-a} = \frac{a}{r} + r \Rightarrow \sqrt{x+a} + \sqrt{x-a} - r = \frac{a}{r} + r - r = \frac{a}{r}$	10