

(۱) $\frac{(1+9+25+\dots+100)}{\sqrt{\quad}} \cdot \frac{(1+4+9+\dots+100)}{\sqrt{\quad}} = \frac{(100)(100)}{100} = 100$ ۱۱ و ۱ ۳۱ و ۶ { ۱ → ۳ } ۲۱ → ۲	۱
الف) $\sqrt[3]{(1+\sqrt{3})^2} \times \sqrt{1+\sqrt{3}} = \sqrt[3]{\frac{(1+\sqrt{3})^3}{1+\sqrt{3}}} = \sqrt[3]{1+\sqrt{3}} = 1+\sqrt{3}$ ب) $\frac{\sqrt{2+\sqrt{5}}}{\sqrt{1+2}} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \frac{\sqrt{2}}{\sqrt{3}} \times (\sqrt{2-\sqrt{5}} - \sqrt{2+\sqrt{5}}) = \frac{1}{\sqrt{3}} (\sqrt{4-2\sqrt{5}} - \sqrt{4+2\sqrt{5}})$ $= \frac{1}{\sqrt{3}} (\sqrt{5-1} - \sqrt{5+1}) = \frac{1}{\sqrt{3}} (\sqrt{4} - \sqrt{6}) = \frac{2-\sqrt{6}}{\sqrt{3}} = -1$	۲
الف) $\frac{\sqrt{8+\sqrt{3}}}{8-\sqrt{3}} - 2(\sqrt{3}-1) = \frac{19\sqrt{3}+19\sqrt{3}}{19} - \sqrt{3} - 1 = \sqrt{3} - 1$ ب) $\frac{2\sqrt{3}-1}{\sqrt{3}+1} + \frac{1}{2-\sqrt{3}} \Rightarrow 2+\sqrt{3}$ $\frac{19-19\sqrt{3}}{19} = 1-\sqrt{3}$ $1-\sqrt{3}+2+\sqrt{3} = 3$	۳
الف) $(\sqrt{1+\sqrt{3}} + \sqrt{3}-1)^2 = 1+\sqrt{3} + \sqrt{3}-1 + 2\sqrt{(1+\sqrt{3})(3-1)} = 2\sqrt{3} + 2\sqrt{2} = 2\sqrt{2}(\sqrt{3}+1)$ $= 2\sqrt{2} + 2\sqrt{2} \Rightarrow \sqrt{2}\sqrt{2} + \sqrt{2}\sqrt{2} = \sqrt{2}\sqrt{2} + \sqrt{2}\sqrt{2}$ ب) $\sqrt{2-\sqrt{3}} = \frac{1}{\sqrt{2+\sqrt{3}}} \Rightarrow \sqrt{2} \times \sqrt{2+\sqrt{3}}$ $\times \sqrt{2+\sqrt{3}} = \sqrt{2}(\sqrt{2} + \sqrt{2}) = \sqrt{4} + \sqrt{4} = \sqrt{4} + \sqrt{4} = \sqrt{4}$	۴
$\frac{3^{91} (1+9+27+81+243)}{3^{91} (1+3+9+27+81)} = \frac{3^{91} \times 324}{3^{91} \times 121} = \frac{3^{91} \times 3^4 \times 3^2}{3^{91} \times 11^2}$ $\frac{3^{91} \times 3^6}{3^{91} \times 11^2} = \frac{3^6}{11^2} = \frac{729}{121}$ $\frac{3^{91} \times 3^6}{3^{91} \times 11^2} = \frac{3^6}{11^2} = \frac{729}{121}$ $\frac{3^{91} \times 3^6}{3^{91} \times 11^2} = \frac{3^6}{11^2} = \frac{729}{121}$ $\frac{3^{91} \times 3^6}{3^{91} \times 11^2} = \frac{3^6}{11^2} = \frac{729}{121}$	۵

$(a-b)^r (a+b)^r = (a-b)^r (a+b)^r \Rightarrow (a^r - b^r)^2 = a^{2r} - b^{2r} = \sqrt{4-r} \cdot \sqrt{4+r}$ $\Rightarrow \sqrt{4-r} = 1 + \sqrt{4+r} \Rightarrow 1 - \sqrt{4+r} = \sqrt{4-r} \quad (\sqrt{4-r})(\sqrt{4+r}) = 1 \Rightarrow (1\sqrt{4-r} - \sqrt{4+r})^2 = 1 \Rightarrow 4 - 4\sqrt{4-r} - 4\sqrt{4+r} + 1 = 0$ $\Rightarrow 5 - 4\sqrt{4-r} - 4\sqrt{4+r} = 0 \Rightarrow 4\sqrt{4-r} = 5 - 4\sqrt{4+r} \Rightarrow \sqrt{4-r} = \frac{5 - 4\sqrt{4+r}}{4}$	6
$a^{\frac{1}{2}} + \frac{1}{a^{\frac{1}{2}}} = 1 + 1 \Rightarrow (a^{\frac{1}{2}} + \frac{1}{a^{\frac{1}{2}}})^2 = 4 \Rightarrow \sqrt{a} + \frac{1}{\sqrt{a}} = 2 \Rightarrow \sqrt{a} = 1 \Rightarrow a = 1$ $\Rightarrow a^{\frac{1}{2}} + \frac{1}{a^{\frac{1}{2}}} = 1 + 1 = 2 \Rightarrow \sqrt{a} = 1 \Rightarrow a = 1$	7
$A = \sqrt[3]{\frac{1}{r}} \times \left(\frac{1}{r}\right)^{-\frac{1}{3}} = r^{\frac{1}{3}} \times r^{\frac{1}{3}} \times r^{\frac{1}{3}} = r = 1$ $(rA)^{-\frac{1}{3}} = \frac{1}{r} = \frac{1}{r}$	8
$a^{\frac{1}{2}} = r \times a^{\frac{1}{2}} \Rightarrow a = r^{\frac{1}{2}} \times a^{\frac{1}{2}} \Rightarrow a = r^{\frac{1}{2}} \Rightarrow (a^{\frac{1}{2}} - a)^2 = 0$ $\Rightarrow a(r^{\frac{1}{2}} - 1) = 0 \Rightarrow \begin{cases} a = 0 \\ a = -\frac{1}{r} \\ a = \frac{1}{r} \end{cases} \quad \frac{\sqrt{r}-r}{\sqrt{r}+1} \times \frac{\sqrt{r}-1}{\sqrt{r}-1} = \frac{-r\sqrt{r}+r}{r} = -\sqrt{r}+1$	9
$\sqrt{4+a} + \sqrt{4-a} = 1 \Rightarrow \sqrt{4+a} = 1 - \sqrt{4-a} \Rightarrow \sqrt{4+a} = 1 - \sqrt{4-a}$ $\sqrt{4+a} - 1 = \sqrt{4-a}$	10