(به دلیل ننوشتن اسم نمره شما صفر لحاظ میگردد)

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$(1!+3!+5!+\dots+95!)(2!+4!+6!+\dots+26!)$

$1,\ 6,\ 0,\ 0 \qquad 2,\ 4,\ 0,\ 0$

$(7)(6)=42$ → ۲

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الف) $\sqrt[4]{\frac{(1+\sqrt7)^2}{4+\sqrt7}} = \sqrt[4]{\frac{8+2\sqrt7}{4+\sqrt7}} = \sqrt[4]{2}$

ب) $\frac{\sqrt{(\sqrt5-1)^2}-\sqrt{(\sqrt5+1)^2}}{2} = \frac{-2}{2} = \boxed{-1}$

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$\frac{1}{\sqrt2}\left(\frac{\sqrt3+\sqrt5}{\sqrt5+\sqrt3}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) = \frac{\sqrt2}{2}\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) = \frac{\sqrt{6-2\sqrt5}-\sqrt{6+2\sqrt5}}{2} =$

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الف) $\frac{2\sqrt6+3\sqrt{30}}{5-\sqrt6} - \frac{2}{\sqrt3-1} = \frac{(2\sqrt6+3\sqrt{30})(5+\sqrt6)}{(5-\sqrt6)(5+\sqrt6)} - \frac{2(\sqrt3+1)}{(\sqrt3-1)(\sqrt3+1)} = \frac{19\sqrt3+19\sqrt2}{19} - \frac{2(\sqrt3+1)}{2}$

$\sqrt3+\sqrt2-\sqrt3+1 = \boxed{\sqrt2-1}$

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ب) $\frac{3\sqrt3-1}{4+\sqrt3}+\frac{1}{2-\sqrt3} = \frac{(3\sqrt3-1)(4-\sqrt3)}{(4+\sqrt3)(4-\sqrt3)}+\frac{(2+\sqrt3)}{(2-\sqrt3)(2+\sqrt3)} = \frac{13\sqrt3-13}{13}+(2+\sqrt3)$

$\sqrt3-1+2+\sqrt3 = \boxed{1+2\sqrt3}$

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الف) $\sqrt{\left(\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}\right)^2}-2 = \sqrt{\frac{1+\sqrt3+\sqrt3-1+2\sqrt2}{\sqrt3-\sqrt2}} = \sqrt{\frac{2\sqrt3+2\sqrt2(\sqrt3+\sqrt2)}{\sqrt3-\sqrt2(\sqrt3+\sqrt2)}}$

$= \sqrt{2(\sqrt3+\sqrt2)^2}-2 = \sqrt2(\sqrt3+\sqrt2)-2 = \boxed{\sqrt6}$

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ب) $2^{\frac{5}{12}}\cdot 3\cdot 2^{\frac{3}{4}}\cdot 2\cdot 2^{-\frac{1}{12}} = 2\times2\times3=12$

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$\frac{3^x(1+3+9+27+81+243)}{2^{x-2}(1+2+4+8+16+32)} = 4\left(\frac{3}{2}\right)^x\cdot\left(\frac{364}{63}\right)=52$

$\frac{4}{9}\times\left(\frac{3}{2}\right)^x\times52=52$

$\frac{4}{9}\times\left(\frac{3}{2}\right)^x=1 \Rightarrow \boxed{x=2}$

$\frac{4}{9}\quad\frac{9}{4}=1$

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$$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2 = \left((a-b)^2\cdot(a+b)^2\right)^2 = (a^2-b^2)^4 =$$

$$(a^2)^4 - 4(a^2)^3(b^2) + 6(a^2)^2(b^2)^2 - 4(a^2)(b^2)^3 + (b^2)^4$$

$$(\sqrt6-2)^2 - 4\sqrt2(\sqrt6-2) + 6(\sqrt2)^2 - 4\sqrt2(\sqrt6+2) + (\sqrt6+2)^2 =$$

$$(6+4-4\sqrt6) - 4\sqrt2(2\sqrt6) + 12 + (6+4+4\sqrt6) =$$

$$32-16\sqrt3 = 16(2-\sqrt3) \rightarrow t=16$$

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$$a+\frac1a = f \;/\; \left(a+\frac1a+\sqrt2\right)^2\left(a+\frac1a-\sqrt2\right)^2 = (f+\sqrt2)^2\cdot(f-\sqrt2)^2 =$$

$$(f^2-2)^2 = \left(a^2+\frac{1}{a^2}+2-2\right)^2 = \left(a^2+\frac{1}{a^2}\right)^2 \Rightarrow \left(\left(\sqrt[4]{7-4\sqrt3}\right)^2 + \frac{1}{\left(\sqrt[4]{7-4\sqrt3}\right)^2}\right)^2$$

$$\left(2-\sqrt3+\frac{1}{2-\sqrt3}\right)^2 = \left(\frac{8-4\sqrt3}{2-\sqrt3}\right)^2 = \left(\boxed{4}\right)^2 = 16$$

$$16 = 2^t \Rightarrow \boxed{t=4}$$

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$$A = \sqrt[5]{4\sqrt[3]{16}}\left(\frac12\right)^{-\frac43} \rightarrow \sqrt[5]{4\sqrt[3]{16}} = \sqrt[15]{2^{10}} = \sqrt[3]{4}, \quad \left(\frac12\right)^{-\frac43} = 2^{\frac43} = \sqrt[3]{16} \Rightarrow \sqrt[3]{64} = 4$$

$$(2A)^{-\frac13} = 8^{-\frac13} = \left(\frac18\right)^{\frac13} = \boxed{\frac12}$$

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$$\sqrt[7]{a} = 3^3 \times a^{\frac{10}{7}} \Rightarrow a = 3^{21}\times a^{10} \Rightarrow a^{16}\cdot 3^{21} = 1 \Rightarrow a^{14} = 3^{-21}$$

$$a^2 = 3^{-3}, \quad a = 3^{-\frac32}$$

$$3^{\frac32} = 3\sqrt3$$

$$\frac{\left(\frac1a-3\right)}{1+\sqrt3} = \frac{3\sqrt3-3}{1+\sqrt3}\cdot\frac{(1-\sqrt3)}{(1-\sqrt3)} = \frac{6\sqrt3-12}{-2} = \boxed{6-3\sqrt3}$$

۲ ۹

$$\sqrt{x+a}+\sqrt{x-4}-2 = A$$

$$2A = x+a-x+4-4 = a$$

$$2\left(\sqrt{x+a}-\sqrt{x-4}\right)$$

$$\boxed{A=\frac a2}$$

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